Introduction to Orbifolds

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My summer research project, supervised by Dr Bryan Wang at the Australian National University, consisted of an introduction to the general theory of orbifolds.

Intuitively, an orbifold is a topological space which is locally homeomorphic to a quotient of some Euclidean space by the action of some finite group of automorphisms. As such, my first task was to understand such a concept; I then went on to learn how this intuition can be translated into solid mathematics. As orbifolds are generalisations of manifolds, the remainder of my project involved generalising standard constructions on manifolds to constructions on orbifolds.

My work stemmed from a paper by Satake [1], who originally defined orbifolds (under the name 'V-manifolds'). In this paper, Satake defines orbifold atlases, smooth maps between orbifolds, bundles on orbifolds (in particular, the tangent bundle), differential forms, and Riemannian metrics.[1] I worked through these definitions and constructions, filling in the (numerous) details which Satake chose to omit. This specifically was much more difficult than initially expected, and consumed the majority of my time.

Orbifolds typically (but not always!) arise as quotient spaces of smooth manifolds under the actions of compact Lie groups. Hence, the study of orbifolds in general is of great importance to the study of manifolds with some degree of symmetry.

The project came with many benefits, not all mathematical: in particular, the experience of working closely with a supervisor has helped to prepare myself for an honours year (and possibly further study). The opportunity to spend time with other enthusiastic mathematics students was also greatly appreciated; it's possible that I learned more from our mealtime conversations than from my project!

Alexander Amenta received a 2010/11 AMSI Vacation Research Scholarhip

References

[1] Ichiro Satake, *The Gauss-Bonnet theorem for V-manifolds*, Journal of the Mathematical Society of Japan (1957)

1 Satake also proves an analogue of the Gauss-Bonnet theorem, and exhibits an application of orbifolds to the study of Siegel modular forms; these topics were beyond the scope of my project.