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Three Dimensional Reconstruction of Coronary Blood Vessels

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1 Introduction

Coronary heart disease is a condition which affects the blood vessels (coronary arteries). It is the leading cause of death in Australia. Partially blocked coronary arteries can decrease heart functions and can cause pain in the chest, arms, neck or jaw. Stroke or heart attacks can be the reason of a person's death or paralysis. Therefore, early diagnosis of heart disease is very important and useful. It is known that, by examining the movement of the coronary arteries, the location of the future blockages can be identified. By examining coronary angiograms, the most likely location of the coronary blockages can be predicted. The problem is, the angiogram gives two dimensional (2D) images. To actually predict the location of the future heart blockages, the three dimensional (3D) images can be a lot more helpful. 3D images of the coronary arteries are easy to interpret as the position, shape and the movements of the arteries are more clear. The aim of this project is creating a 3D image of a heart as it moves through its cardiac cycle from the 2D images of the angiograms of a heart.

2 Background of the Data

The data is collected by Dr. Aiden O'Loughlin (University of Western Sydney) from coronary angiograms of a patient's heart. Two positions can be found in the cardiac cycle which give the most stretch. These two positions are when the heart is filled with blood, which is called end-diastolic phase and the blood has pumped out of the heart, which is the end-systolic phase. Four different angiogram images are selected from two different camera positions. From these four images, we collected the eleven vessel points. These points are measured in pixels. These angiogram images are shown in figure 1.

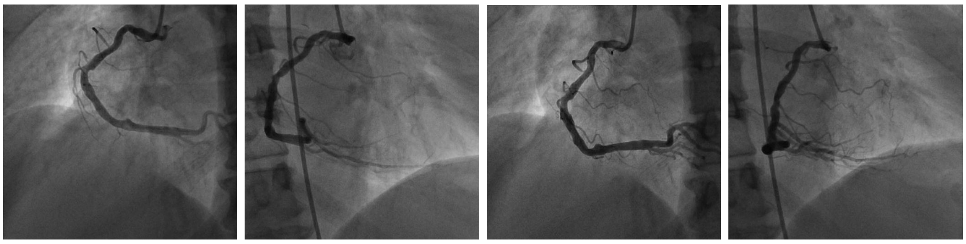

Figure 1: 2D images of a heart's end-diastolic phase (first two from left) and end-systolic (ES) phase (last two from left) with two different cameras.

3 3D Reconstruction

There are different methods that can be used to reconstruct a 3D model. To construct a 3D model, the fundamental matrix and the essential matrix calculation plays an important role. We examined two methods, eight point method and five point method. The additional aim for this project is to consider the limitations of each method.

3.1 Fundamental and Essential Matrix

The fundamental matrix is a 3×3 matrix which gives us the relationship between the two image points to its 3D projective image. The fundamental matrix is of rank two. Therefore, at least seven corresponding matching points are needed to solve the fundamental matrix. For two corresponding image points, the fundamental matrix is denoted by F, can be calculated using the relation:

$$\vec{u}_i^T F \vec{v}_i = 0 \tag{1}$$

Here, $\vec{u}_i$ and $\vec{v}_i$ are two 2D homogeneous matching points from two cameras. We can rearrange this matrix as a vector $\vec{f}$, where $\vec{f}$ is a vector containing the nine entries of F.

The essential matrix properties are very similar to the fundamental matrix. When the cameras are calibrated, the essential matrix needs to be calculated from the fundamental matrix. For a given set of points, we can only get a unique essential matrix whereas, we can get multiple fundamental matrix. The relationship between the fundamental matrix and the essential matrix is:

$$E = K'^T F K \tag{2}$$

Here, K and K' are the calibration matrices.

3.2 Eight Point Method

Longuet Higgins [1] introduced the eight point algorithm for essential matrix calculation. Eight corresponding coordinate points are needed to evaluate this method. The algorithm includes calculating the fundamental matrix with eight corresponding coordinate points. The problem with the eight point algorithm is, it can be quite sensitive to any error present in the data [2]. Additionally, if more than eight points are used, the system is over determined and needs to perform a least square approximation. In our project, we evaluated the eight point algorithm method.

3.3 Five Point Method

Five point algorithm has the advantages over eight point algorithm as this method needs less points [3]. This method is easy to work with. The difference with the eight point method arises while calculating the essential matrix. For this method, several additional constraints can be used.

4 Outline of the 3D Reconstruction Method

3D reconstruction process is highly appreciated and needed in computer graphics, virtual reality and medical science. The 3D reconstruction process needs to follow several steps. The steps are given below:
1. Computation of fundamental matrix from point correspondence.
2. Computation of camera matrices from the fundamental matrices.
3. Computation of the point in space that projects to the image points correspondence.
The accuracy of the reconstruction depends on the accuracy in each step.

4.1 Computation of the Fundamental Matrix

The first step for 3D reconstruction is calculating the fundamental matrix. The fundamental matrix can be calculated from a set of point correspondences. The fundamental matrix is useful with the uncalibrated images. Using equation 1 , it is possible to construct a set of linear equations of the form: $A\vec{f} = \vec{0}$ where, $\vec{0}$ is a zero vector and $\vec{f}$ is a vector which contains the nine entries of the fundamental matrix F. The solution vector $\vec{f}$ is defined up to a scale. By solving this equation, we get the equation matrix A which is a 9×9 matrix.

4.2 Computation of the Camera Matrices

The camera matrix is a 3×4 projection matrix which gives us the mapping of a pinhole camera from the 3D points in the world to 2D image. If x represents the 3D point in homogeneous coordinates and y represents the image point, then the relationship will be: $y = Px$. The general form of a camera matrix is:

$$P = \begin{bmatrix} f & 0 & p_x & 0 \\ 0 & f & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Here, f is the focal length and (p_x, p_y) is the principal point coordinates. We take the first camera matrix, $P = [I|0]$ as an identity matrix with extended zero column entries at the end. Therefore, the first camera matrix has focal length, $f = 1$ and the principal point coordinates, $p_x = 0$, $p_y = 0$. To compute camera matrices, one needs to calculate the calibration matrix and the essential matrix. The calibration matrix is of the form:

$$K = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix}$$

When the cameras are calibrated, the essential matrix needs to be calculated from the fundamental matrix and the calibration matrix [4].The calculation of the singular value decomposition (SVD) of E is needed in this manner. Let, the SVD of E be, $U\Sigma V^T$. To calculate the second camera matrix, we need to define new matrices such that:

$$W = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}; W^{-1} = W^T; Z = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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Here the matrix Z has the property: $Z = W\Sigma$. Once the essential matrix is known, the camera matrices can be retrieved from the essential matrix. With the SVD of the essential matrix, two factorizations are possible where: $[t]_\times = VW\Sigma V^T$ and $R = UWV^T$ or UW^TV^T. Here, $[t]_\times$ is the matrix representation of the cross product with the translation matrix t. This can be written as:

$$[t]_\times = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix}$$

Therefore, we will have four possible choices for the second camera matrix P'[5] such as:
$P' = [UWV^T| \pm u_3] or, P' = [UW^TV^T| \pm u_3]$
Here, u_3 is the last column of U.

4.3 Computation of the 3D Points

Computation of the 3D points is the last step of the 3D reconstruction. There are several ways to compute the 3D points in space. In our project, we used two different methods. These two methods are linear triangulation method and rotation-shift decomposition method. Both methods are described in the following sections.

4.3.1 Reconstruction Using Linear Triangulation

The linear triangulation is an efficient method to find the 3D points. This method is the direct analogue of the DLT method. For each of the corresponding points we have, $x = PX$ and $x' = P'X$. Let, x be the image point $(x, y, 1)$ and P^{iT} is the i_{th} row of the camera matrix P. From the matrix multiplication of $[x]_\times PX$, we can construct an equation of the form, $AX = 0$ where,

$$A = \begin{bmatrix} xP^{3T} - P^{1T} \\ yP^{3T} - P^{2T} \\ x'P'^{3T} - P'^{1T} \\ y'P'^{3T} - P'^{2T} \end{bmatrix}$$

A is a 4×4 matrix. Hence, the solution is the least eigenvector of A^TA. The solution gives us the 3D homogeneous point. The calculation for $AX = 0$ is very similar to the working for the fundamental matrix.

4.3.2 Reconstruction Using Rotation-Shift Decomposition

Using the rotation-shift decomposition method, we calculate the rotation and the translation (shift) using the essential matrix. The formulas to find the rotation and translation matrices are:
$R = UW^TV^T$ and $[t]_\times = V(W\Sigma)V^T$.
If the two normalized image points are $y = (y_1, y_2)$ and $y' = (y_1', y_2')$, then the 3D point, $x = (x_1, x_2, x_3)$ can be calculated using the following formulas:

$$x_3 = \frac{(r_1 - y_1'r_3)t}{(r_1 - y_1'r_3)y} \qquad or, \qquad x_3 = \frac{(r_2 - y_2'r_3)t}{(r_2 - y_2'r_3)y} \tag{3}$$

Here, r_i is the i_th row of the rotation matrix. Other two 3D coordinates can be found from x_3 with the following formula:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_3 \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \tag{4}$$

Hence, we get the Cartesian 3D point $x = (x_1, x_2, x_3)$ with the rotation-shift decomposition method.

5 Results

With all the results we get using different methods, we plotted 3D coordinates and get the images for reconstructed points:

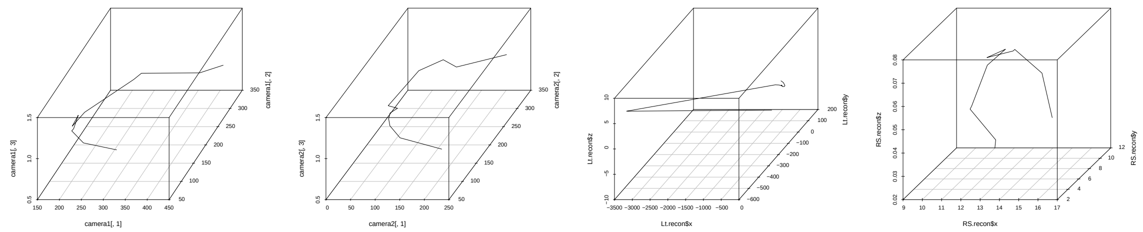

Figure 2: Camera1 and camera2 raw data images (from left) and reconstructed 3D image of a heart with linear triangulation method (third from left) and with rotation-shift decomposition method (right image).

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These images are similar to the raw data images. However, reconstructed images are somehow rotated and shifted. The reconstructed images are from a different angle. Once we get the 3D points, we can project back these points to compare with the actual raw points. The comparison with the projected back points are explained with the following images.

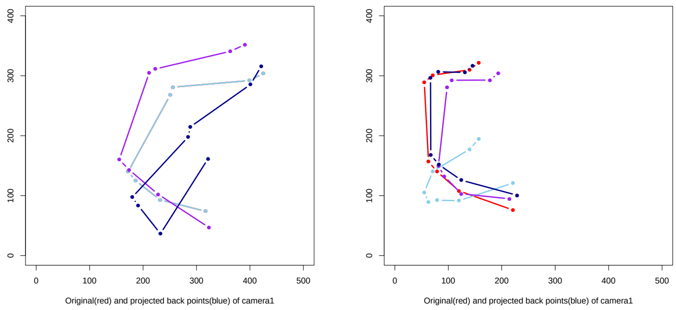

Figure 3: Comparison with the projected back points with camera1 (left) and camera2 (right). Red represents the original raw data, blue represents the projected back points using 5 point method, purple represents the projected back points using 8 point method (linear triangulation) and skyblue represents the projected back points using 8 point method (rotation-shift).

From the above images, we can see the 8 point method with linear triangulation gives the similar shape of the raw data although the points are not exact. On the other hand, 5 point method gives very accurate results for camera 2, but not for camera 1. Although the rotation shift method gives exact points for camera1 (cannot see the red points as it is exactly the same as the skyblue points) but it is not a very good approximation for camera2. Our aim was to reconstruct a 3D model of a heart to look into the shape of the arteries. As long as the shape is preserved, it can be used to predict future blockages. Although the points are not exact, 8 point method using linear triangulation preserves the shape of the arteries. Therefore, 8 point method is more robust to the errors than 5 point method.

6 Reconstruction Error

In the process of the 3D reconstruction method, the main errors come from the data. The data points were calculated manually which can be a logical source of errors.

This is a very common problem as there is no way to find points in the image without any error. Although, the 8 point method gives very good approximation, the differences between the projected back points and the actual points are not very small. Further research needs to include this error.

7 Conclusion

Our aim was to reconstruct the 3D model of a heart. We used the 8 point algorithm and 5 point algorithm to find the 3D points for both of the phases. The reconstructed 3D coordinates are plotted to get the reconstructed images. The 8 point method gives more accurate results than the 5 point method for our data. This can be the first step towards creating a 3D model of a patient's heart where the future artery blockages can be identified. However, further research needs to be done to improve the results and to calculate the actual points.

8 Acknowledgement

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References

[1] Higgins, L.: A computer algorithm for reconstructing a scene from two projection. Nature **293** (1981) 133–135

[2] Hartley, R.I.: In defense of the eight-point algorithm. IEEE Pattern Analysis and Machine Intelligence (1997)

[3] Li, H., Hartley, R.: Five-point motion estimation made easy. In: Pattern Recognition, 2006. ICPR 2006. 18th International Conference on. Volume 1., Ieee (2006) 630–633

[4] Nistér, D.: An efficient solution to the five-point relative pose problem. Pattern Analysis and Machine Intelligence, IEEE Transactions on **26**(6) (2004) 756–770

[5] Hartley, R., Zisserman, A., ebrary, I.: Multiple view geometry in computer vision. Volume 2. Cambridge Univ Press (2003)