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MATHEMATICS

Probability Modelling

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The aim of my project was to understand the definition and construction of the Dirac operator. I mainly followed the construction given in “Dirac Operators in Riemannian Geometry” by Thomas Friedrich.

The Dirac operator is a first order differential operator that can be defined on any manifold admitting a spin structure. It is named after Paul Dirac who considered it, in the 1920s, while trying to create a new relativistic theory of the electron. This theory had much success in explaining previously unexplained phenomena and made predictions which were later verified. Some of the central ideas in this theory, although motivated by physical considerations, also turned out to be of mathematical importance especially in finding interrelations between topology, geometry and analysis.

The construction of the Dirac operator involves a number of steps.

- The construction the appropriate of Clifford algebra.
- The derivation of the groups Spin and SpinC and their representations from this Clifford algebra.
- Determining the right map between these groups and the special orthogonal group.
- Defining a spin structure on the manifold using the groups and map.
- Calculating the appropriate connection which in turn defines the Dirac operator.

Doing this required a lot of background knowledge, particularly concepts from differential geometry such as principal bundles, structure groups and connections in principal bundles.

I enjoyed studying this topic and will probably continue to study it in my honours year.

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See http://www.ice-em.org.au/students.html#scholarships2007