Brownian motion

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A Markov process is *recurrent* if for any given point in its state space, it returns infinitely often to that point with probability one. The focus of my AMSI project was the recurrence properties of random walks and their limit processes, with particular emphasis on Brownian motion. I began by reading up on the classical theory of random walks in d-dimensional integer lattices, as well as some more recent research on random walks on plane tilings and circle packings. Wolfgang Woess' book-length summary of the subject was particularly interesting.

The later part of the project involved investigating a family of random walks in the complex plane, with each walk defined by a stationary distribution over the nth roots of unity. Using some well-known results on cyclotomic fields, I showed that each of the walks can be embedded in an integer lattice, with the dimension dependent on n. Consequently, the walks are recurrent if and only if they range over a periodic lattice. For the remaining walks, a theorem of K. L. Chung shows that they return to every open neighbourhood of a point infinitely often with probability one. I also used the bivariate central limit theorem to establish that the walks converge to Brownian motion.

Unfortunately, I was unable to extend this approach to prove the recurrence or transience of more general random walks in the complex plane. The essential problem was the non-existence of bases for general modules.

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See: www.ice-em.org.au/vs09.html