Modelling the Stochastic Growth of Ornate Rock Lobsters (*Panulirus Ornatus*)

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My project involved exploring different stochastic models for the growth of Ornate Rock Lobsters (*Panulirus Ornatus*). Ornate rock lobsters are tropical spiny lobsters that support commercial and local islander diver fisheries in the Australian and Papua New Guinean waters of Torres Strait. Each year, over 900 tonnes or approximately $17 million (AUD) worth of ornate rock lobsters are trawled from the waters. As a result, current research has been focused on modelling the complicated population dynamics of these lobsters and conducting stock assessments to ensure the sustainability of this fishery. Another motivation for this project is that in current literature, the growth of sea animals is usually taken to be a smooth function of time or age. However, this is not suitable for modelling crustacean growth because lobster growth is determined by moulting and hence is discontinuous in nature. Thus, it is important to develop models which encapsulate these stochastic growth properties. In particular, we consider the growth of ornate rock lobsters to be modelled via two stochastic processes. The first process being the time between moults, and the second process is the corresponding length increment of growth after moulting.

Our dataset was collected from 74 lobsters (42 females and 32 males) in a three year laboratory study. The variables that were recorded include: the lobster's unique identification number, its sex, its carapace length (defined as the distance from the tip of the lobster's head to the beginning of its tail section), the water temperature, and the date the observation was made. The other variables of interest, moult increment and moult interval, were calculated by determining the change in carapace length and date respectively.

The first model we explored involved applying survival analysis techniques to describe the relationship between moult interval and premoult length. Specifically, we treated lobster moulting as the event of death or failure, and used a Cox Proportional Hazards model.

Our hazard function is,

$$h(t|PL) = h_0(t) \cdot \exp(\beta_1 PL + \beta_2 PL^2 + \beta_3 TEMP).$$

The hazard function specifies the instantaneous rate of moulting given that the lobster has survived up to a certain premoult length. Our model involves the first and second order premoult lengths (indicated as PL and PL^2) and water temperature (indicated as TEMP) as covariates. The water temperature was included to be a variable because previous studies have shown evidence for water temperature to have a significant effect on rock lobster growth. Our model is dependent on sex, and the individual lobster ID was used to account for individual variability.

The $\boldsymbol{\beta}$ model parameters were estimated using the statistical software R, and all covariates were tested to have a significant effect on moult interval based on a 5% significance level. Furthermore, model assumptions such as proportionality of hazards function, absence of influential data points and a linear relationship between log hazards function and the covariates, were validated using Schoenfeld, martingale and DfBeta residual values.

The next model we explored was the linear mixed effects model. It was used to describe the relationship between moult increment and premoult length. The model is described with the following equation,

$$INC_{i,t} = \zeta_i + \beta_1 PL_{i,t} + \beta_2 PL_{i,t}^2 + \beta_3 (PL * SEX)_{i,t} + \beta_4 (PL^2 * SEX)_{i,t}$$
$$+ \beta_5 TEMP_{i,t} + \beta_6 (TEMP * SEX)_{i,t} + \beta_7 SEX_{i,t} + \epsilon_{i,t},$$

where $INC_{i,t}$ is the moult increment of lobster i at time t, PL and PL^2 are the first and second order premoult lengths respectively, SEX is the gender indicator of lobster i, TEMP is the water temperature, and $PL * SEX, PL^2 * SEX$ and $TEMP * SEX$ are the interaction terms between the first and second order premoult lengths and temperature against sex respectively.

$\epsilon_{i,t}$ is the error term and it is Normally distributed with 0 mean and variance σ_ε^2. ζ_i describes the random effect term which accounts for individual variability, and it has a Normal distribution with mean 0 and variance σ_{PL}^2. The random effect term is independent of the error term. Also, all the covariates in the model were allocated to be fixed effects.

The $\boldsymbol{\beta}$ model parameters were estimated using the statistical software R. Even though only PL and $PL * SEX$ were tested to be statistically significant at the 5% significance level, we decided to retain all the covariates in our model.

For model diagnostics, a quantile-quantile plot was conducted to test the assumption of Normality of our data and a residuals plot was applied to test the assumption of constant variance among the residuals. An increasing trend of variance among the residuals was observed, and the right tail of the quantile-quantile plot greatly deviated away from a straight line trend. These observations suggest that some degree of caution must be taken when we use the linear mixed effect model for predictions.

An alternative approach to describe the relationship between premoult length and moult interval is through a variation of the model used by Millar and Hoenig (Millar et al., 1997). For our sex-specific model, we let INT_{PL} denote the intermoult period as a function of premoult length.

The distribution of intermoult period is modelled with a Log-Normal distribution, that is, $\log(INT_{PL})$ is Normally distributed with mean ν_{PL} and variance σ^2. The mean function is described by,

$$\nu_{PL} = \log(a) - \frac{\sigma^2}{2} + b \times PL.$$

This formula was chosen because the expected intermoult period would then become an exponentially increasing function of premoult length which is a property frequently exploited for the expected intermoult period of crustaceans.

The model parameters were estimated using maximum likelihood methods. This model assumes the lobsters to moult independently and asynchronously; this assumption is justified in past studies. The limitations of this model include, it does not take account of the temperature variable or correlations due to repeated measurements of individual lobsters. Also, although the variance term accounts for lobster variability, the variance function is constant for all moult lengths, and this makes our model rather rigid.

Another model used to describe the relationship between moult increment and premoult length is an adaptation of Russo et al.'s model (Russo et al., 2009). We modelled the moult increment as a Gamma process with shape parameter $(L_\infty - L_t)(1 - e^{-k(INT)})$, scale parameter λ and drift parameter γ; where L_t is the premoult length at time t, L_∞ is the maximal length that is attained as time approaches infinity, k is the growth parameter and INT is the mount interval. Furthermore, the moult increment follows a gamma distribution with parameters described with

$$INC \sim \Gamma((L_\infty - L_t)(1 - e^{-k(INT)}), \lambda).$$

These model parameters were chosen because the expected moult increment would then follow a *von Bertalanfy Growth Function*, which is one of the most common deterministic equations used for describing animal growth.

Our model is sex-specific and all parameters were estimated using maximum likelihood methods. The limitations of this model include: it does not take into account the temperature factor or correlation due to repeated measures. The variance function is strictly decreasing, which makes the model rather rigid and unable to account for non-monotonic behaviour of variance that is often observed in real data. Also, because our Gamma likelihood function assumes time independence, this results in the loss of critical moult interval information from the observed data.

The goal of this project was to develop models to describe and predict the growth process of the ornate rock lobster population at any given time. Such growth models can be formed through the composition of the two studied stochastic processes, the moult interval and moult increment process. To achieve our goal, Monte Carlo simulation techniques were used to generate sample rock lobster growth curves; and for a large enough sample size of generated growth curves, taking the average of them would represent the expected lobster population growth curve.

Two combinations of growth models were simulated. The first growth model involved the superposition of the Cox proportional hazards model for describing moult interval, along with the linear mixed effects model for describing moult increment. The second growth model was a composition of the stochastic log-Normally distributed moult interval model, along with the stochastic Gamma process for describing moult increment.

Simulations from the log-Normal-Gamma model predicted that for lobsters of both sexes to moult more frequently and with smaller moult increments when compared to the Cox proportional-mixed effects simulations. On the other hand, the Cox proportional-mixed effects model predicted the growth rate of both male and female lobsters to decrease after approximately 2000 days and that a maximum length will eventually be attained. This phenomenon from the Cox proportional-mixed effects model simulation agrees with real world observations, suggesting that this is a suitable model to describe ornate rock lobster growth.

Despite the various limitations for both growth models, it appears that both models are valid to describe the growth of ornate rock lobsters. However, the Cox proportional-mixed effects model appears to be the better model as it demonstrates more of the observed real world phenomenon of lobster growth.

This project has helped broaden my knowledge of statistical models and software packages. Through this project, I have gained a range of skills such as the ability to organise and extract maximal information from messy datasets, the experience in writing a scientific report and talk slides, as well as gaining confidence at public speaking. The Big Day In seminars allowed me to acknowledge the wide variety of mathematical topics currently under research, and it provided me the opportunity to meet with industry professionals and socialise with other fellow mathematicians. I would like to thank my supervisor Prof. You-Gan Wang, the University of Queensland, AMSI and CSIRO for their support.

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