Assessment of the effects of viscous fluid shear stress on an elastic membrane

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Introduction

Study of fluid-structural interaction (FSI) has gained considerable interest in the last decade. The procedure involves multi-physics coupling of force and displacements between a fluid and structural solver respectively. FSI is generally classified into 1-way or 2-way depending on the nature of feedback between the solvers. In 1-way FSI, physical properties are mapped from the analysis of a fluid-model to the solid-model. Force is transferred from fluid to the solid model only once as an external load. Interaction is one-directional as no responses are feedback to the fluid solver. However, 2-way FSI, mapping is performed in an iterative loop in which pressure from fluid is transferred to the solid model and the displacement of solid model is transferred back to the fluid model. An iterative process is performed until coupling convergence is achieved.

In this project we have modeled a 2-dimensional section of an arbitrary artery where low density lipoprotein (LDL) is transported and subsequently accumulates on an artery wall. Our main objective on this project is to formulate the physics of the problem and model this with corresponding partial differential equations (PDEs) and appropriate boundary conditions (BCs); the model is then setup in the commercial PDE solver ANSYS Fluent. The fluid medium is blood and comprises a mixture of particles. Of interest are the LDL particles that are transported passively with the bulk fluid. As they are transported within the bloodstream, a portion may be deposited along the artery wall some and subsequently causing an inflammatory response which may result in the formation of an atherosclerotic plaque (C.X.Chen, 2012). Further growth of the lesion at the wall results in reduced arterial cross-sectional area which eventually increases the wall shear stress. This leaves the plaque vulnerable to rupture and activate subsequent blood clotting, which may result in risk of heart attack, stroke and peripheral vascular disease.

Since the biochemical and mechanical events that cause plaque growth are difficult to assess in a laboratory setting, computational approaches to model these events have grown to become the popular alternative. Current literature has also suggested that low wall shear stress (WSS) strongly correlates with accumulation of LDL concentrations (C.X.Chen, 2012). It has also been explained that the elevated shear stress is occurred at the entrance section of stenosis and induce rupture of plague (CJ Slager, 2005). In this paper, the FSI model is established and the Wall shear stress is observed for the rigid and the deforming wall.

Methods

The 2D model of the artery is designed in ANSYS v14.5 as shown in figure 1. 2D model is constructed over 3D to eliminate set-up variables as well to reduce the number of elements that ultimately reduce the computational time and memory usage.

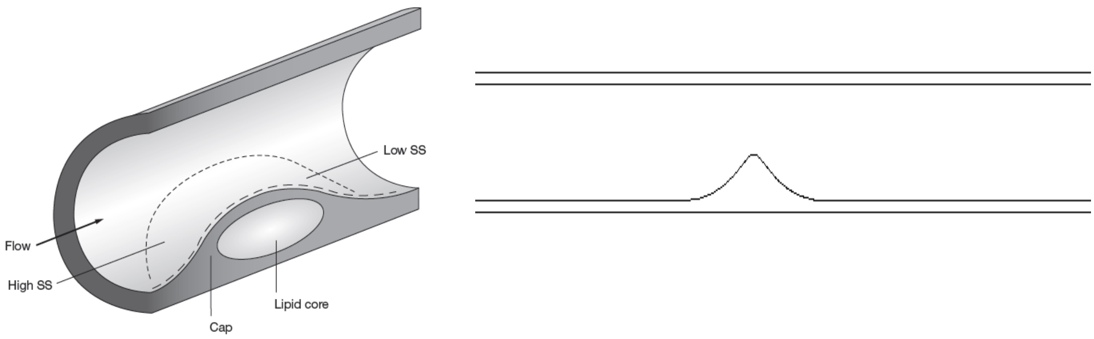

Figure 1: Cross-section view of artery with plaque in (a) 3D model (CJ Slager, 2005) (b) 2D model.

The 2D model is restrained by applying the boundary condition. The boundary conditions for the model are as shown in Figure 2.

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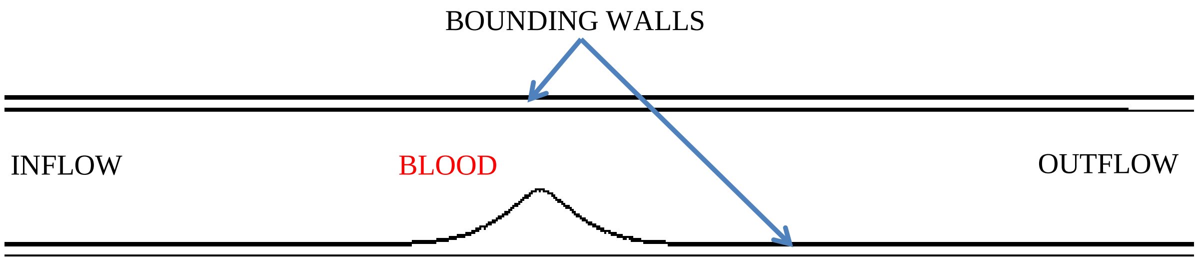
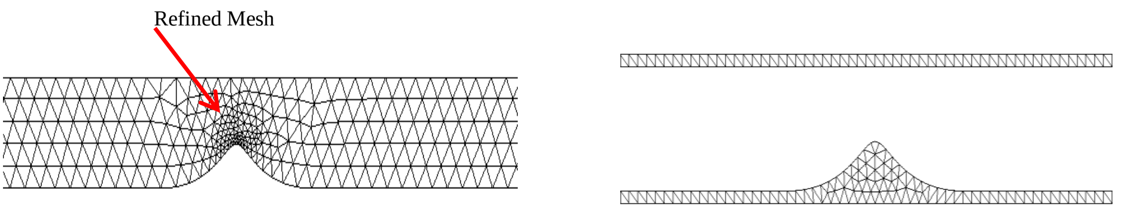


Figure 2: Model outline

Small section of the artery is modeled to analyze the fluid flow. Thickness of the wall is assumed to be 2mm and the diameter is 5mm. Fluid medium is blood in our case. No-slip condition is assumed for the wall and the flow is assumed to be laminar. After finalizing the model shown in figure 2, the next step is to generate mesh. Meshing is to divide the model into finite elements. It is very essential to know the concept behind the meshing and its significances. Large number of cell will produce accurate results but it is strongly dependent on the imposed limitations dominated by the computational costs and calculation turnover times. Various shapes of the mesh can be overlaid depending on what the structure comprises.

Unstructured mesh type is used for our fluid and structural interface as it allows the calculation of flows in or around geometrical features of arbitrary complexity in lesser time to generate mesh and mapping. Mesh refinement along the areas such as near the boundary layer, high gradient and flow reversal are much simpler performed with unstructured meshes.



(a) Fluid Domain (b) Solid Domain

Figure 3: Triangular unstructured mesh for the fluid and the structural domain of the artery model

Table 1: Fluid properties

Density [kg/m^3]	**Dynamic viscosity [kg/m/s]**
1060	0.0035

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For the fluid model, a parabolic velocity profile was set up at the inlet. Since the heart pumps up the blood in a pulsatile fashion our fluid motion is also set as a pulsatile flow to match the physics as well as to reduce the computational expenses by eliminating the extra distance fluid covers to let the velocity profile fully develop as it passes through the channel. The expression to obtain the parabolic velocity profile is shown below.

$$U = U_{max}\left(1 - \frac{r^2}{R^2}\right)(\alpha + \beta \sin(\omega t)) \qquad 1$$

Where $U_{max} = \frac{3}{2}u_{avg}$, R is the artery radius and r is the reference distance from the centerline.

The outlet is set as pressure dependent variable which is the function of mass and momentum. Average static pressure is set for the outlet boundary condition. It is set to the zero gauge pressure which is the operating condition for our setup. Outlet relative pressure allows the pressure profile at the outlet to vary based on upstream influences while constraining the average pressure to a user-specified value ($\bar{p}_{spec}$).

Where $\bar{p}_{spec} = \frac{1}{A}\int_s \; p_{ip}.dA$ and p_{ip} is the imposed pressure at each integration point and the integer is evaluated over the entire outlet boundary surface.

For our structural part, boundary conditions are set. The walls are assigned to be linear –elastic material. However, artery shows hyper-elastic behaviour physiologically but to simplify our model and to reduce computational variables it is set as a linear elastic material. The boundary conditions for the artery are as shown below:

Table 2: Material setup for structural solver

Density [kg/m^3]	**Young's Modulus [MPa]**	**Poisson's ratio**
1000	3	0.499

Inner part of the top and bottom wall which is in contact with the fluid is set to system-coupling and the fluid is deforming. Dynamic mesh setting in Fluent is the key setting to allow 2-way FSI. Deformed shape of the model is being computed by the finite-element code ANSYS Mechanical and is being transferred to fluent. The

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deformation vector is defined for each individual node that makes up the coupled surface.

Multiple studies have shown that the coupled problems can be subdivided into two classes, monolithic and partitioned. Monolithic approach treats the coupled problem as one system of equation that describes the interaction of both fields hence steps ahead simultaneously in time. However, partitioned approach solves each field separately and communicates the interface to connect the components. Partitioned approach demonstrates number of strategies to couple the model however gives greater flexibility to solve the problem (Gatzhammer, 2008). Partitioned approach is used for our FSI model. Fluid Model and the structure model are separately set and then coupled to transfer force from fluid and displacement from the structure until convergence is achieved.

Fluid Physics

Blood is used as our fluid medium which is considered as incompressible and Newtonian. Research had shown that blood can be considered as a Newtonian fluid under certain condition due to the rheological properties of red-blood cell-plasma suspension. Although, it was shown that Newtonian assumption is reasonable (Shipkowitz et al., 1998) for large arteries studies. The fluid (blood) properties are assumed to change over the time therefore it is set to unsteady flow (also known as transient). Conservation of mass for unsteady flow states that "rate of increase of mass within control volume equals to the net rate at which mass enters the control volume''. Mathematically the mass conservation for our case can be expressed as:

$$\frac{dm}{dt} = \sum_{in} \dot{m} - \sum_{out} \dot{m} \qquad 2$$

Formulating this for an incompressible fluid, the equation can be modified as follows:

$$\nabla \cdot U = 0 \qquad 3$$

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Conservation of the momentum for the fluid flow can be achieved by force balance and is generally called the Navier-stokes equation. It helps to obtain the velocities of fluid elements at each location:

$$\underbrace{\frac{\partial u}{\partial t}}_{unsteady\ term} + \underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}}_{convective\ term} = - \underbrace{\frac{1}{\rho}\frac{\partial p}{\partial x}}_{pressure\ gradient} + \underbrace{v\frac{\partial^2 u}{\partial x^2} + v\frac{\partial^2 u}{\partial y^2}}_{diffusive\ term} \quad 4$$

The above equation describes the transport of fluid velocity changing over time on an Eulerian reference frame. It explains that as the fluid gains energy it starts to travel until some opposing force, viscosity diffuses the momentum of the fluid. Principle of conservation of momentum is incorporated with the convection and diffusion that affect the fluid flow. It involves normalizing the mathematical equation to yield the nondimensional governing equations (Tu, 2008) with governing dimensionless parameter, the Reynolds number:

Reynolds number: $Re = \frac{VL\rho}{\mu}$

Where V is the average velocity, L a characteristic length scale, ρ density and μ viscocity. Reynolds number influences the convective and diffusive terms of the Navier-Stokes equation.

One of the important features of the dimensionless equations (Reynolds Number) is the flow behavior to be the same for different fluids for instance water and air even though they differ on physical properties such as density, velocity and viscosity. As a result the numerical results of dimensionless governing equation, X-momentum and Y-momentum, are obtained to be identical.

Structural Physics

The study of structure mechanics involves the theory and methods that are necessary to obtain the deformation, stresses, vibration and stability of the structure. The application of structural physics is immense and has been applied to components of different scale. Engineers and the scientists are analyzing the structures from micro to large industrial applications. Studies of structure on micro scales include analyzing the mechanics of the cell and its deformation.

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Finite element software has been largely used for the structural analysis. It involves discretization of geometries into series of small elements for instances for the artery wall which may deform in any direction can be discretized into 3D-geometry. We have assumed that our artery is made up of linear elastic material that has a property of stress-strain relationship in the form of small deformation and linear relationship. Dynamic analysis of the structure is performed as the arteries show large stresses and displacements. Behaviour of structure mechanics is described using Lagrangian approach. Lagrangian view fixes the frame of reference to the object observed. The observer is bound to the object of interest and follows its movement. We are interested in displacement and the displacement is obtained by the motion of node and the motion is characterized by Lagrangian approach.

Explicit and Implicit method are two basic types of dynamic analysis. We have used the implicit method over explicit to solve our structural solver. Explicit method relies on the previous time step. It precedes the next step on the basis of information that has been passed on from the earlier step. It accumulates the truncation error from each time step, as a result the final solution is over estimated. It is generally good for the fixed time step. Implicit method relies in the current time step (ΔT) only therefore this method eliminates the problem of accumulating truncation error.

Equation of motion is used to describe the behaviour of the motion and time. For our dynamic analysis the equation is represented as shown below:

$$\underbrace{[M]\,\{\ddot{U}\}}_{inertial\ term} + \underbrace{[C]\{\dot{U}\}}_{damping\ term} + \underbrace{[K]\{U\}}_{spring\ term} = \underbrace{[F]}_{external\ work} \qquad 5$$

Equation of motion is discretized implicitly to obtain displacement after the load is applied:

$$\begin{aligned} &u_{t+\Delta t} = u^{*}_{t+\Delta t} + \beta \Delta t^{2} \ddot{u}_{t+\Delta t} \\ &M\ddot{u}_{(t+\Delta t)} + (1+\alpha)\left[c\dot{u}_{(t+\Delta t)} k u_{(t+\Delta t)} - F_{(t+\Delta t)}\right] - \alpha\left(c\dot{u}_{(t)} + K u_{(t)} - F_{(t)}\right) = 0 \\ &\beta = \frac{1}{4}(1-\alpha)^{2} \end{aligned} \qquad 6$$

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The β factor is case dependent to stabilize the solution which is the function of damping coefficient (α). It is essential to obtain the correct value of β so that the solution is converged. Computationally, it was challenging to acquire the precise value of beta. β Factor was varied for implicit structural solver until a stable range was found so brute force testing was employed to identify a satisfactory range of values.

In the process of stabilizing, the mesh deformation on Eulerian solver was used on the basis of diffusion based smoothing (solving Laplace's equation on the mesh).

$$\nabla.(\gamma \nabla \vec{u}) = 0$$
$$\gamma = \frac{1}{V^{\propto}} \quad , \quad \propto \geq 0 \qquad 7$$

Where, $\vec{u}$ is the mesh velocity and V is the normalised cell volume

The displacement equation is discretized using finite volume method and the node positions are updated according to the equation as shown below.

$$\vec{x}_{new} = \vec{x}_{old} + \vec{u}\Delta t \qquad 8$$

The solution is initiated by educated initial conditions. Solution will have periodic properties which coincident with inflow boundary condition. After certain time, stable boundary closed-curve is observed which explains the stable solution period. Solutions may diverge in several conditions which will not provide the results therefore the setup of the numerical solution is to check again until convergence is achieved.

Results and discussion

The velocity contour as shown in Fig.4 clearly explains the difference between the rigid and deforming wall. Pulsatile flow is set up for our flow condition and the pattern of the flow for one cycle is as shown below. It can be noted that the velocity

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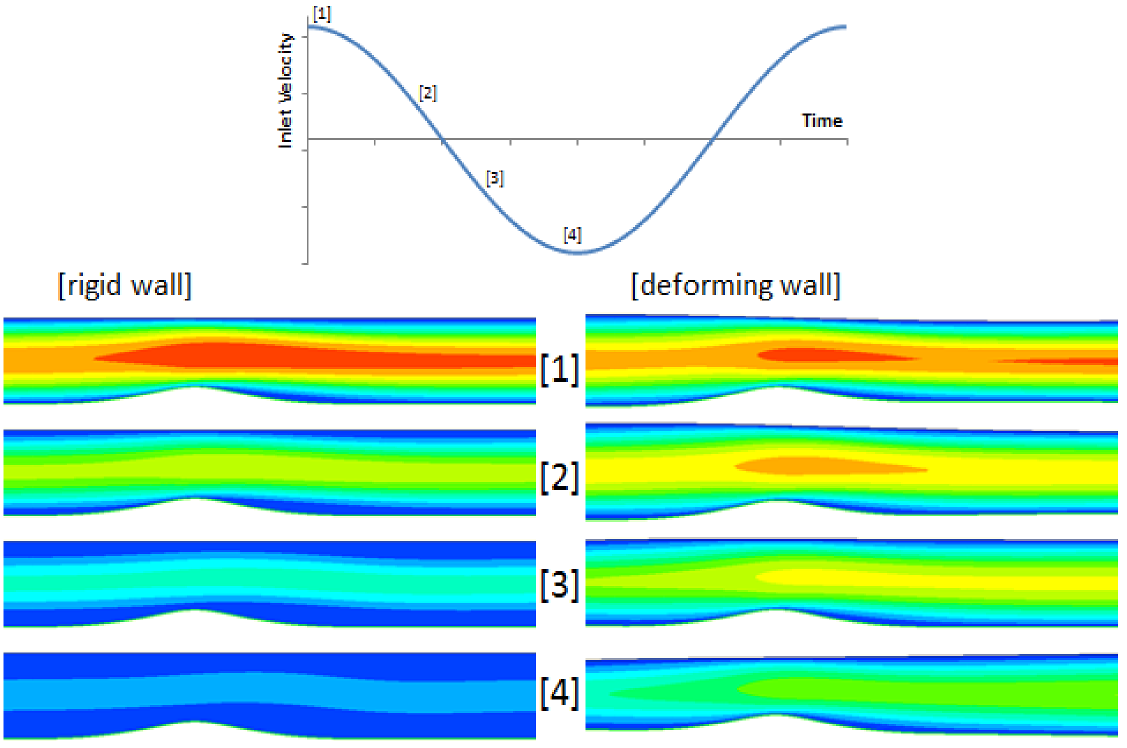
varies in each time step. Area associated with the lump is characterized with high velocity. It can be explained due to the reduction in diameter, flow gain momentum to travel in higher speed as a result higher velocity is experienced in the narrow region.



(a) Rigid wall *(b) Deforming wall*

Figure 4: Variation of the velocity contour at different time step between (a) rigid wall and (b) deforming wall

Wall shear stress is also compared between the fixed and deforming wall. As expected from the velocity contour plot the rigid wall has higher wall shear stress in the stenosis region. Shear stress is directly proportional to the velocity gradient therefore the higher the velocity the higher the wall shear stress.

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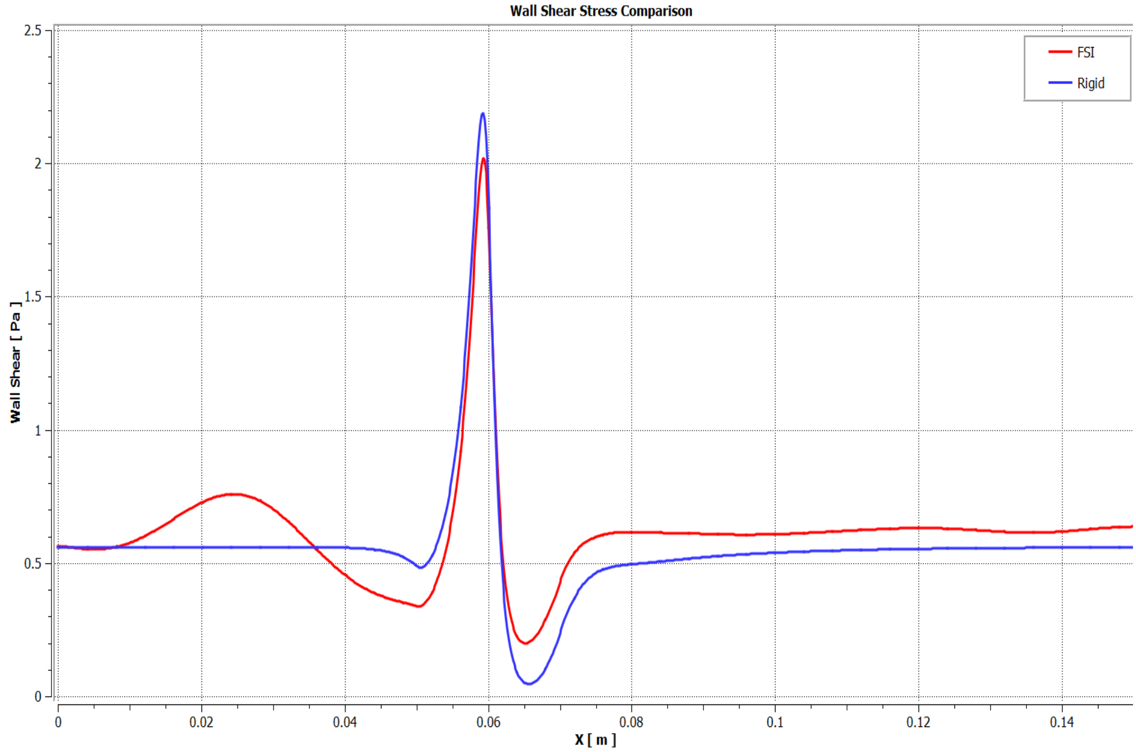

Figure 5: Variation of wall shear stress between rigid and deforming wall

It has been suggested that the time-averaged wall shear stress for both rigid and the deforming wall are identical and only minor differences were observed (Jonas Lantz, 2011).

Conclusion

The validation of FSI model for the rigid and deforming wall was performed. Low WSS in an artery is closely associated with the phenomenon of high LDL concentration. To accurately model LDL accumulation, it is necessary to capture the WSS well. Rigid wall showed the higher WSS compared to the deforming wall. WSS varies significantly for a deforming vessel wall relative to a rigid wall, during a given time period. Further studies are to be employed to validate the physics model with established test data (refer to Turek FSI benchmark model) as well as to employ species transport to model LDL accumulation on arterial wall as influenced by wall shear stress.

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