A literature review of multi-objective programming

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Abstract

This paper provides a survey of methods that have been developed to solve multi-objective problems. We focus only on non-interactive exact methods that generate the entire set of optimal solutions. The basic concepts of these multi-objective problems are introduced along with their solutions. We then review some of the available exact methods.

1 Introduction

Multi-objective programming is used in application for many real world problems including problems in the fields of engineering, mining and finance.

In multi-objective programming there are multiple conflicting objectives whereby improving one objective will reduce the value of others, leading to a trade-off between solutions. It is assumed that no single solution will optimise all objectives simultaneously because this would be a trivial case.

The main aim of multi-objective programming is to assist a decision maker (DM) to choose a preferred solution amongst all the trade-offs. In this case, it is not necessary to generate all solutions when the DM is involved in the process since some solutions may be eliminated at each stage. However, in this paper we will focus on non-interactive exact methods that do not involve the DM in order to generate the entire solution set.

Multi-objective problems take the form of linear (MOLP), integer (MOIP), and mixed integer (MOMIP) which have continuous, discrete, and both continuous and discrete solutions respectively. Due to the nature of MOMIP, there are several different types of problems. In this paper we will focus mainly on bi-objective mixed integer programming problems and the generation of only extreme supported non-dominated solutions for general MOMIP problems. There are many studies that deal with MOIP and MOLP problems independently, but there is a lack of literature for their hybrid, MOMIP. Due to the nature of their solutions, MOIP and MOLP cannot be directly used to solve MOMIP.

This paper focuses on compiling and summarising articles published in the English language for MOLP, MOIP and MOMIP problems, with additional updates of recent developments since surveys like Ehrgott and Gandibleux (2000) and Ruzika and Wiecek (2005). It should be assumed that the algorithms generate all non-dominated solutions in the objective space unless stated otherwise. There are many reasons as to why the more recent algorithms solve in this objective space instead of the decision space. The objective space is typically much smaller than the decision space because in almost every case there are fewer objectives than decision variables. This simplifies the problem and it becomes less computationally demanding. Efficient solutions in the decision set were also proved to frequently map onto the same solution of the outcome set by Benson (1995), leading to redundant solutions in the decision set. Algorithms that solve in the decision space like Steuer (1986), Armand and Malivert (1991), Armand (1993) and Sayin (1996) are hard to apply practically as computational demands increase substantially as problem size increases.

In Section 2 the general formulation and concepts of multi-objective optimisation problems are outlined. In Section 3 a classification scheme is defined for clustering the approaches in Section 4. The most popular methods that are used in the review will be clarified in Section 3 to avoid repeating explanations in Section 4.

2 Properties of multi-objective optimisation problems

A multi-objective mixed integer programming problem is formulated as:

$$\min Cx = (f_1(x),\ f_2(x), \dots, f_p(x))^T$$
$$subject\ to\ x\ \in X,$$

where $X = \{Ax \leq b, x \geq 0, x^C \in\ \mathbb{R}^n$ and $x^I \in \mathbb{Z}^n\}$ is the set of all feasible solutions. The solutions x^c and x^I denote the set of feasible solutions for multi-objective linear and integer problems respectively. $A \in \mathbb{R}^{m\times n}$ is an $m \times n$ matrix of the m constraints and n decision variables and $b \in \mathbb{R}^m$ is the corresponding right hand side.$C \in \mathbb{R}^n$ is a $p \times n$ matrix that represents the $p \geq 2$ objective functions to be minimised. The outcome set of the solutions is defined as:

$Y = \{y \in \mathbb{R}^p : y = Cx, x \in X\}$

Set X and Y are known as the decision space and objective space, respectively.

The following notation is used for $y^1, y^2\ \in \mathbb{R}^p$:

$$y^1 \leq y^2 :\Leftrightarrow y^1 \leqq y^2 \text{ and } y^1 \neq y^2,$$
$$y^1 \leq y^2 :\Leftrightarrow y_k^1 \leqq y_k^2, \forall k = 1, \dots, p$$

The Pareto cone is defined as:

$$\mathbb{R}^p_{\geqq} \coloneqq \{y \in \mathbb{R}^p : y_k \geq 0, k = 1, \dots, p\}$$

Consider points $x, x^* \in X$. A feasible solution $x^* \in \mathbb{R}^n$ is called efficient/Pareto optimal if there exists no x such that $Cx \leqq Cx^*$. The outcome of x^*, y^* is then called non-dominated. If $y_k^* <\ y_k$ for all k, then y_k^* strictly dominates y_k. Otherwise, y_k^* is weakly non-dominated. A supported non-dominated solution is a Pareto solution that is the optimal solution to the weighted sum single-objective problem:

$$min\ \left(\lambda_1 f_1(x) + \dots + \lambda_p f_p(x)\right)$$

If an existing efficient solution cannot be found by solving the above problem, it is an unsupported non-dominated solution.

Let Y_{ND} denote the set of non-dominated points. For any $y \in Y_{ND}$, y^{conv} denotes a convex combination of all the non-dominated points, excluding y. There can be three types of non-dominated points. They are:

- Extreme supported if and only if there exists no $y^{conv} \leqq y$

- Non-extreme supported non-dominated if there exists no $y^{conv} < y$ but there exists $y^{conv} = y$
- Unsupported non-dominated if there exists $y^{conv} < y$

3 Classification of approaches

The approaches within each problem type will firstly be grouped by the methods which they employ and then chronologically. There are other methods for solving problems, e.g. Lin, Zhu and Sheng (2003) use homotopy to find efficient solutions for convex multi-objective programming. However, only the most popular methods used in the approaches reviewed in this paper are listed below in this section.

- Weight space decomposition refers to the partitioning a weight set and solving the weighted sum single-objective problem.
- The ϵ-constraint method was first introduced by Haimes, Lasdon and Wismer (1971). This method chooses a single-objective to be optimised while every other objective is treated as a constraint. The extreme point that is computed is then used to determine the bound on the objectives, and this is repeated until there are no new solutions left.
- The branch and bound method uses a tree structure and each node branches out into sub-problems until all sub-problems are solved.
- Duality refers to the relationship between the primal and dual LP $\min\{p^T x: Ax \geq b, x \geqq 0\}$ and $\max\{b^T u: A^T u \leq p, u \geqq 0\}$. If both the primal and dual LP are feasible, they both have optimal solutions x' and u' that satisfy $p^T x' = b^T u'$, i.e. the minimal element of the primal is equivalent to the maximal element of the dual in the outcome set.
- The two phase method by Ulungu and Teghem (1995) involves finding all supported non-dominated solutions by solving the weighted sum problem in the first phase and finding all other non-dominated solutions in the second phase, usually using enumerative techniques which develop search spaces from the supported non-dominated solutions found in the first phase.

4 Approaches

4.1 Linear Programming

In MOLP problems, all objectives are linear and must be optimised over a convex polyhedron. MOLP problems are solved as subproblems for MOIP and MOMIP and all non-dominated solutions of MOLP are supported. MOLP problems are popular and there is a lot of literature that covers finding the efficient set, some of which are covered in this section.

Benson (1998a) generates an outer approximation algorithm in the outcome space. The main advantage of this algorithm is that there is no need for backtracking or bookkeeping which is needed when solving in the decision space, as in Benson (1997). This method is later implemented into Benson (1998b) which introduces a hybrid vector maximisation approach which was first introduced by Kuhn and Tucker (1951). Benson

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incorporates the special simplicial partitioning technique used by Ban (1983) and Tuy and Horst (1988) into the outcome space using outer approximation. First, a point p that lies in the interior of the efficiency-equivalent polyhedron Y is created along with its simplex, S°, and vertex set, $V(S^{\circ})$. At the k^{th} iteration, the algorithm examines the set of all vertices, $V(S^k)$ of the current compact polyhedron S^k that contains Y. The algorithm terminates if each element of $V(S^k)$ belongs to Y because $S^k = Y$. Otherwise, a new polyhedron, $S^{k+1} \subset S^k$ that contains Y, is created by adding a linear equality to S^k. The vertex set $V(S^{k+1})$ is then computed and the algorithm continues until $S^k = Y$ is satisfied. Later in Benson (1998c) it was found that this algorithm also generated weakly efficient points in the outcome space.

Benson and Sun (2000) proved that a feasible basis for the linear program LP(w)

$$\max\ (w)^T Cx$$
$$subject\ to\ x \in X,$$

The weight set

$$W^0 \equiv \{w \in \mathbb{R}^p \big| w_j > 0, j = 1,2,\dots,p\}$$

can be decomposed into a finite union of subsets with a one-to-one correspondence between the weights and efficient extreme solutions in the outcome space. Using this result, Benson and Sun (2002) develop a weight set decomposition algorithm. At each step k, a weight w^k is chosen and the LP is solved for an optimal extreme point solution. Each subsequent weight is found by initialising a global tree search method. If no more weights can be found then the algorithm terminates.

Ehrgott, Puerto and Rodrigues-Chia (2007) use the scalarisation theorem and single-objective duality theory to develop a new algorithm. This algorithm focuses on applications in network optimisation problems. Luc (2011) introduces two approaches to duality, one based on the duality relationship between minimal and maximal elements of a set and its complement, and another using polarity between convex polyhedral sets and the epigraph of its support function. Improvements to existing duality relations are also explored. Ehrgott, Lohne and Shao (2012) use geometric duality theory to derive a dual variant of the algorithm of Benson (1998b). This method constructs the dual extended image instead of the primal image. Once the dual image is obtained, geometric duality is used to obtain the primal image.

Ida (2005) uses an extreme ray generation method to sequentially generate efficient points and rays. This is done by adding inequality constraints to the polyhedral feasible region. In the algorithm, objective values for each extreme ray are obtained and tested for efficiency. A new efficient ray is generated if the pair of extreme rays have efficient solutions when one of the efficient solutions is eliminated in the row process step. All efficient extreme rays and points are obtained when all the rows have been checked.

Krichen, Masri and Guitouni (2012) generate maximal efficient faces using adjacency between efficient extreme points. This algorithm explores efficient extreme points and uses simplex pivots to find adjacent vertices of the current extreme point. Initially an efficient extreme point is found and at each iteration thereafter, combinations of these points are generated to define frontiers of the efficient faces.

4.2 Integer Programming

The main difference between MOLP and MOIP problems is that MOIP objectives are discrete, not continuous. The introduction of integer variables allows for feasible solutions that no longer lie on a line segment. This leads to the existence of non-supported efficient solutions which are much harder to find. Some of the methods used in finding these non-supported are explained here.

In bi-criteria problems, it is well known that $2|N| - 1$ subproblems must be solved to generate all non-dominated solutions, where N is the non-dominated set. Initially $|N|$ subproblems are solved to generate all points in N, and then $|N| - 1$ more subproblems are solved to make sure that there are no more non-dominated points that exist between the ones that have already been generated. Laumanns, Thiele and Zitzler (2006) used an adaptive ϵ-constraint method and showed that problems of higher dimensions require a bound of $\mathcal{O}(|N|^{m-1})$, where m is the number of objectives. Dächert and Klamroth (2013) develop an algorithm that needs to solve at most $3|N| - 2$ subproblems for tri-criteria problems.

Przybylski, Gandibleux and Ehrgott (2010a) generalise the two-phase method and apply it to the tri-objective assignment problem. Appropriate lower and upper bounds are computed and used to update the initial search space from the first phase. Any upper bounds that are dominated are removed and any non-supported non-dominated point that is found is inserted into the updated search space. The approach of Dächert and Klamroth (2013) use is similar to this, but filter out redundant search areas.

Due to the issues the classical ϵ-constraint method had with finding weakly efficient solution, Mavrotas (2009) introduced an augmented ϵ-constraint method which augments the objective function using the weighted sum of extra slack or surplus variables. This method was improved by Mavrotas and Florios (2013) specifically for MOIP problems and required fewer subproblems to be solved. Zhang and Reimann (2013) develop a variation of the augmented ϵ-constraint method called the simple augmented ϵ-constraint method which improves through using an acceleration algorithm with an early exit and an acceleration algorithm with bouncing steps. The algorithm of Özlen and Azizoğlu (2009) recursively solves problems with lesser objectives using the ϵ-constraint method. The objective functions are minimised and maximised to generate the ranges for the non-dominated set which are then used to solve for all non-dominated solutions. The algorithm was later improved by Özlen, Burton and MacRae (2013). Kirlik and Sayin (2014) find the non-dominated set using a search space of $(p - 1)$ dimensions. This algorithm uses rectangles in the search space, with the initial rectangle covering the $(p - 1)$ dimensional space. Each rectangle is defined using lower and upper bounds. These lower and upper bounds are found by minimising and maximising each objective function, respectively. The rectangles are partitioned into smaller disjoint rectangles and this is repeated until there are no rectangles left to search.

Klein and Hannan (1982) propose a sequential generation method for finding all non-dominated solutions in the decision space. This method solves a sequence of progressively more constrained single-objective integer problems. At each step a new constraint is added which excludes previously generated efficient points. This allows points which are dominated by the generated non-dominated solutions to be eliminated. A variation of this method is used by Sylva and Crema (2004) which sequentially solves weighted sum problems instead of single-objective problems and later, Sylva and Crema (2007) propose another variant that finds a well-dispersed subset of non-dominated points. An improvement of the algorithm by Sylva and Crema (2004) is developed by Lokman and Köksalan (2012) which decreases the number of additional constraints to be added at each step.

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Lemesre, Dhaenens and Talbi (2007) propose parallel partitioning method (PPM) to solve bi-objective problems. This method uses three stages to determine the entire Pareto front. Firstly, the problem is solved for extreme solutions to limit the search space. Next, the search space is divided up by searching the efficient solutions. Lastly, the solutions found from the previous stage are used to find any other efficient solutions. An extension of this method for any number of objectives is done by Dhaenens, Lemesre and Talbi (2010).

4.3 Mixed Integer Programming

MOMIP problems are the hybrid of MOLP and MOIP problems. There are several types of problems within MOMIP itself due to the combination of continuous and integer variables. So far, there is no existing algorithm that can solve for mixed 0-1 integer programs with $p > 3$ objectives and no general algorithm to find all non-dominated solutions. There is a lack of literature for MOMIP problems but some of the methods that do exist are covered here.

Mavrotas and Diaokoulaki (1998) modify the single-objective branch and bound algorithm to find efficient solutions in mixed 0-1 MOLP problems in the decision space. Initially all binary variables $x \in \mathbb{R}^n$ are considered free variables relaxed to $x \in [0,1]^n$ and at the following branch of the combinatorial tree an additional binary variable will become fixed until eventually all combinations are found and the MOLP problem with the fixed binaries are solved. The corresponding non-dominated points to these nodes are stored and updated in D_{ex}. Dominated points are removed from D_{ex} and non-dominated points are added. Later, Mavrotas and Diaokoulaki (2005) further extend to find the efficient solutions of this problem using a vector maximisation approach of the branch and bound method. This algorithm was found to be missing some efficient solutions by Vincent (2009) and Vincent, Seipp, Ruzika, Przybylski and Gandibleux (2013) who then correct the work of Mavrotas and Diakoulaki (2005) in the bi-objective case and explain the issues of the algorithm. Jozefowiez, Laporte and Semet (2012) propose a general multi-objective branch and bound method which does not iteratively solve single-objective problems. The lower and upper bounds are defined as sets of points in the objective spaces instead of being single values. Stidsen, Andersen and Dammann (2014) use branch and bound to find all non-dominated solutions for bi-objective mixed integer problem where all integers must be binary and only one of the objectives may be a continuous. This algorithm first solves the problem with all binary values as free variables. Branching is done on the relaxed binary variables and in each node, a six-tuple of values are saved. The algorithm will try to fathom a solution from the six-tuple until there are none left to fathom.

Przybylski, Gandibleux and Ehrgott (2010b) develop some additional properties for the weight space for MOMIP and develop their algorithm based on this. The algorithm utilises the bi-objective algorithms of Cohon (1978) and Aneja and Nair (1979) and recursively reduces multi-objective problems into bi-objective problems which can then solved by the bi-objective algorithms. Özpeynirci and Köksalan (2010) utilise some of the properties found in Przybylski et al. (2010b) to find all extreme supported non-dominated solutions in general MOMIP problems of any objectives. This method introduces dummy points into the weight space decomposition. A sufficiently small ϵ is chosen to guarantee that one of the dummy points will minimise the resulting objective function if any weight is close to zero. Adjacent points are used to determine

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boundaries of the weight space decomposition and at each iteration, new extreme supported non-dominated points or convex combinations are found until they have all been identified.

5 Summary

This literature review serves as an overview of the research that has been done in solving problems of MOLP, MOIP and MOMIP. This literature review is not a complete review as there is ongoing research for each of these problems and time constraints did not allow for a full review for every problem.

6 Further Research

There are many more methods and approaches that have not been included in this survey and a more complete version would be beneficial to eventually developing a general algorithm to solve for all efficient solutions in MOMIP problems.

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