INTERNATIONAL CENTRE OF EXCELLENCE FOR EDUCATION IN MATHEMATICS

Computational Optimal Control

Ryan Loxton, Department of Mathematics & Statistics, Curtin University of Technology

Optimal control problems are a class of dynamic optimization problems that arise in a variety of engineering, biological and economic applications. The essence of an optimal control problem involves choosing a control function such that a dynamical system and constraints are satisfied and a cost functional is maximized or minimized. The cost functional typically represents revenue, profit or yield if it is to be maximized; or cost, effort or energy if it is to be minimized.

There are well known theoretical results characterizing the solutions to such problems. However, in practice, optimal control problems are generally too complicated to solve analytically. Hence, it is important to develop efficient and robust techniques for solving these problems numerically.

In my project, I investigated one such technique called control parameterization. The basic idea of control parameterization is to subdivide the time horizon into subintervals and then express each component of the control as a constant function in each subinterval. Then the heights of these constant functions enter the problem as parameters with respect to which the objective function is to be optimized. Thus, we reduce the problem into one of optimization over a finite number of parameters, as opposed to the original 'infinite-dimensional' optimal control problem, which involved finding the value of a control function at an infinite number of points. This approximate problem is essentially a mathematical programming problem, which can be solved using well-known techniques. If the number of subintervals is large, then the solution obtained should be a good approximation to the true solution.

I solved a number of practical optimal control problems using a software package called MISER3. MISER3 performs the control parameterization technique described above on a given optimal control problem. It was discovered that while this technique is a good one and is relatively easy to understand and implement, it also suffers from some limitations. Current research in the area is focused on eliminating these limitations.

In addition, some of the problems I encountered required transformation via the Control Parameterization Enhancing Transform (CPET). This 'transforms' the optimal control problem into a different form; this equivalent problem is then implemented and solved by MISER3. Utilizing this technique, improved solutions were obtained in many cases.

In conclusion, I found this vacation scholarship to be very rewarding. It gave me an insight into mathematical research and provided me with inspiration for my honours project. I would definitely recommend that students undertake a vacation scholarship next year if given the opportunity.

Ryan received an ICE-EM Vacation Scholarship in December 2005.
See www.ice-em.org.au/students.html#scholarships05