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AMSI SUMMER VACATION SCHOLARSHIP

Localised Analysis of an Ad Hoc Mobile Network

Author:
Ellen MUIR

Supervisors:
Prof. Peter TAYLOR
Dr. Mark FACKRELL

Collaborators:
Giles ADAMS
Peter BRAUNSTEINS

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1 Introduction

1.1 Background

Ad-hoc mobile networks are self-configuring networks of mobile handsets, connected via wireless links. All mobile handsets, or nodes, are free to move within the network and frequently change their links to one another. For such a network to function properly, it is essential that nodes are willing to act as transit nodes. That is, they forward traffic unrelated to their own personal benefit through the network.

One of the primary challenges related to such networks is determining how to manage information in order to ensure that traffic is efficiently routed through the network. There is also the issue of how to provide incentives to individuals to ensure they behave in a socially desirable manner by routing traffic through the network.

Some of the key advantages of ad hoc mobile networks are that they enable decentralised communication, they do not require heavy investment in telecommunications infrastructure and they are highly flexible. These networks could be desirable for developing countries unable to afford conventional telecommunications infrastructure. They could also be used when the telecommunications infrastructure of a region has sustained damage.

1.2 Model

An ad hoc mobile phone network can essentially be modelled as a graph in which vertices represent individual nodes and edges represent potential routes within the network. The network which was primarily considered is illustrated on the following page.

The model design and notation used is outlined below.

- Let J be the set of all nodes
 - Each node $j \in J$ is a fixed point in the Cartesian plane
 - Each node $j \in J$ is connected to every other node within a specified range (56m was used in simulations)
- Let R be the set of all routes
 - Each route $r \in R$ is an ordered subset of J
 - Each pair of nodes $i, j \in J$ with $i \neq j$ is connected by a route $r \in R$

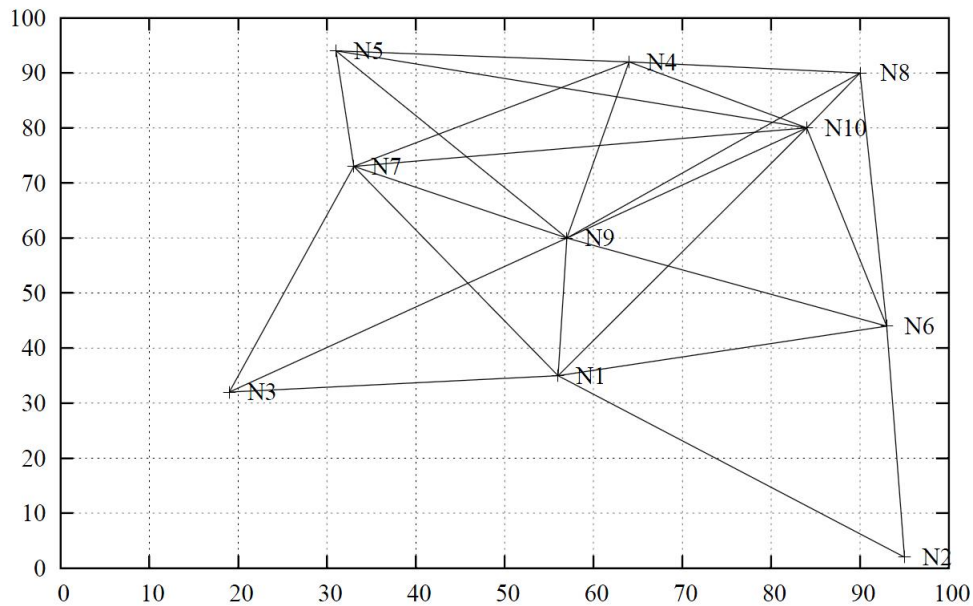


Figure 1: Our 10 node test network.

- Each route $r \in R$ is chosen so as to minimise Euclidean distance

Figure 1 shows the ten node test network used in *Matlab* simulations. This is the same network used by Crowcraft, Gibbens, Kelly and Östring in their 2004 paper.

- Model network traffic or data as flows along the routes within the network
 - The source node uses power initiating the flow
 - Transit nodes use power both receiving the flow and then retransmitting it
 - The destination node uses power receiving the flow
- A utility function is required to quantify the level of satisfaction each node achieves by participating in the network
 - Utility is earned only by the source node for each flow
 - Utility associated with a given route is a function of the flow along that route
 - Utility functions are strictly monotonically increasing and convex
- Flows within the network are constrained by available power

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- Each node $j \in J$ has a power capacity P_j
 - Power spent transmitting a flow is vy_r where
 - * v_s, v_t and v_d denote power per unit flow for being a source, transit and destination node respectively
 - * y_r is the flow along route r

1.3 Notation

Some further notation that is required is outlined below.

- For a route $r \in R$:
 - $s(r)$ is the source node
 - $t(r)$ is the set of transit nodes
 - $d(r)$ is the destination node
- Denote the set of routes for which $j \in J$ is the source, a transit or the destination node by $R^s(j)$, $R^t(j)$ and $R^d(j)$ respectively
- For a route $r \in R$, the utility received by node $s(r)$ is given by the utility function $\mathcal{U}_r(y_r)$

1.4 Numerical Data

The following values were selected for use in the model when completing *Matlab* simulations.

- Power costs
 - $v_s = 1$
 - $v_t = 2$
 - $v_d = 1$
- Power capacity
 - $P_j = 500 \forall j \in J$

The utility function $\mathcal{U}_r(y_r) = \log(y_r)$ was also used in simulations.

2 Static Approach

Assume initially that all users within the network are altruistic. That is, each user seeks to maximise total utility within the network. Krzesinski, Latouche and Taylor (2011) utilised the model described and proposed the following static system:

$$\max_{y_r} \sum_{r \in R} \mathcal{U}_r(y_r) \quad (1)$$

subject to:

$$y_r \geq 0 \quad \forall r \in R \quad (2)$$

$$\rho_j \leq P_j \quad \forall j \in J \quad (3)$$

$$\text{where } \rho_j = \sum_{r \in R^s(j)} \nu_s y_r + \sum_{r \in R^t(j)} \nu_t y_r + \sum_{r \in R^d(j)} \nu_d y_r.$$

This is a non-linear maximisation of a strictly concave function over a convex region. Thus, a unique solution exists. The problem can be solved utilising the Lagrange method and relevant Karush-Kuhn-Tucker (KKT) condition.

The Lagrange method is a non-linear optimisation technique. Let ϵ_j denote the Lagrange multiplier for the power constraint at $j \in J$ and η_r denote the Lagrange multiplier for the non-negativity condition on y_r .

The relevant KKTa conditions are:

$$\begin{aligned} \rho_j &\leq P_j \\ y_r &\geq 0. \end{aligned}$$

The KKTb conditions are:

$$\begin{aligned} \exists \epsilon_j \geq 0 \text{ such that } \epsilon_j (P_j - \rho_j) &= 0 \\ \exists \eta_r \geq 0 \text{ such that } \eta_r y_r &= 0. \end{aligned}$$

2.1 Static Solution

Provided $y_r \leq P$ the static system has the following solution:

$$y_r(\epsilon) = \Theta_r(\nu_s \epsilon_{s(r)} + \nu_s \epsilon_{d(r)} + \sum_{j \in t(r)} \nu_t \epsilon_j) \quad (4)$$

where $\Theta_r(\epsilon)$ is the inverse of the derivative of $\mathcal{U}_r(y_r)$.

2.2 Static Solution - Problems

The main issue with solving the network in such a manner is that the solution is non-distributive. It utilises global information regarding the structure of the network and this information will not be available to all users in real applications.

The solution is relatively easy to compute for the simple test network (see *Figure 1*). However, as additional nodes are added, the computational intensity of the problem rapidly increases.

Finally, the static solution is not dynamic. If any changes occur within the network (for example, if a small adjustment is made to the network topology), then the static solution will become invalid. A dynamic solution that could adapt to such changes is more desirable.

3 Dynamic Approach

An alternative approach is to consider the network as a dynamical system in which each node emits a ‘price signal’. Other nodes use this information to update their data flows so that the system converges towards the optimal solution.

A desirable property of a dynamic solution is that each node uses only locally available information (price signals and knowledge of local network topology) to update their flows. Other important properties are that the dynamical system is stable and has desirable convergence properties.

3.1 Dynamic Solution

The proposed dynamical solution is a regime in which each node updates their price on the basis of local demand

$$\frac{\partial \epsilon_j(t)}{\partial t} = \kappa \epsilon_j(t) \frac{\rho_j(t) - P}{P}. \quad (5)$$

Here, $\rho_j(t)$ is the power being consumed at node j at time t . Note that $\epsilon_j(t)$ denotes the ‘price signal’ emitted by node j at time t . Using the same function

$$y_r(t) = \Theta_r(\nu_s \epsilon_{s(r)}(t) + \nu_d \epsilon_{d(r)}(t) + \sum_{j \in t(r)} \nu_t \epsilon_n(t))$$

for flows as in the static solution, we see that as the utility function is concave, Θ_r is strictly decreasing. Therefore, a higher price at a given node implies less flows will be sent to that node. Similarly, a lower price at given node implies more flow will be sent to that node.

3.2 Dynamic Solution - Optimality

To see that the proposed dynamic system converges to the optimal solution, start by observing the following:

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- If $\rho_j(t) - P_j < 0$ then $\rho_j(t) < P_j$ and this implies that $\frac{\partial \epsilon_j(t)}{\partial t} < 0$. Thus, prices decrease.
 - If $\rho_j(t) - P_j > 0$ then $\rho_j(t) > P_j$ and this implies that $\frac{\partial \epsilon_j(t)}{\partial t} > 0$. Thus, prices increase.

Intuitively, if a node is under-utilised then its price will fall until its full power capacity is utilised. Similarly, if a node is being over-utilised, then its price will rise until the power capacity restriction is met at the node.

Thus, assuming that $\lim_{t \rightarrow \infty} (\mathbf{y}, \boldsymbol{\epsilon}) = (\mathbf{y}^*, \boldsymbol{\epsilon}^*)$, it is clear that $(\mathbf{y}^*, \boldsymbol{\epsilon}^*)$ are optimal flows and prices respectively.

3.3 Dynamic System - KKT Conditions

It is also clear that $(\mathbf{y}^*, \boldsymbol{\epsilon}^*)$ satisfy the KKT conditions. To see this, recall

$$\frac{\partial \epsilon_j(t)}{\partial t} = \kappa \epsilon_j(t) \frac{\rho_j(t) - P}{P}$$

and note that for stable prices ($\frac{\partial \epsilon_j(t)}{\partial t} = 0$), it is required that $\epsilon_j(t) = 0$ and/or $\rho_j(t) = P$. Thus, the necessary KKT conditions are satisfied and the socially optimal solution is a fixed point of the proposed dynamical system. Further, the proposed dynamical solution is a system with a single fixed point (by construction).

3.4 Dynamical System - *Matlab* Simulations

In order to confirm that the proposed dynamical system behaved as predicted, *Matlab* simulations were conducted. Simulations were completed using the simple test network (see *Figure 1*). The numerical solution computed using the dynamic system was then compared to results computed using the static solution by Krzesinski, Latouche and Taylor (2011).

In order to achieve this, the simple test network (see *Figure 1*) was coded into *Matlab* and a simple algorithm which computed all the routes in the network was written. All data flows were initially set to zero and an initial price of 0.1 was set at each node. The differential equation for updating prices within the network was discretised and the simulation was run until the network converged.

It was found that the numerical results for the dynamical solution matched those obtained by Krzesinski, Latouche and Taylor (2011) for the

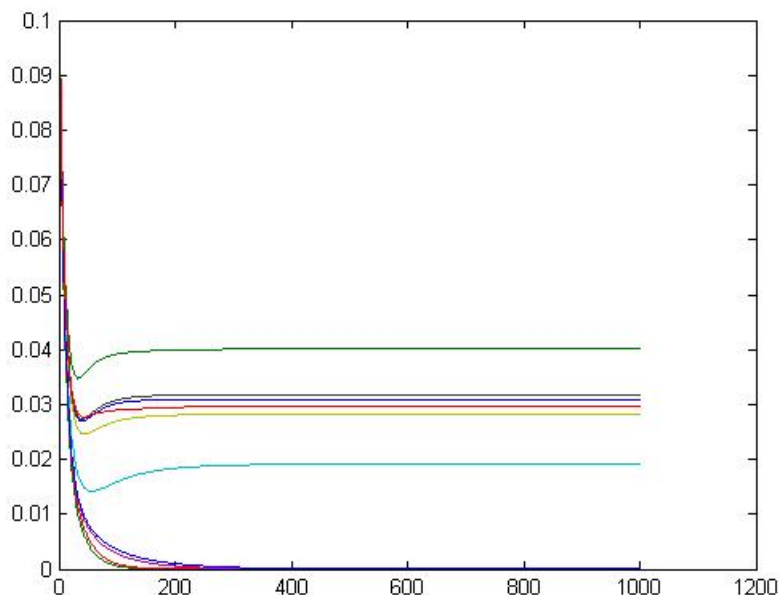


Figure 2: Plot of **prices** at each node over time.

static solution. From *Figures 2,3* and *4*, it can be seen that the dynamic solution also displayed desirable convergence properties. In each of the figures, the relevant timescale is simulation iterations.

3.5 Dynamic Solution - Network Topology Experimentation

After successfully completing the initial simulations, further experimentation with the dynamical solution was conducted. An important property of the dynamical solution is that the system has the ability to adapt to changes to the network topology. Thus, simulations were run in which new users were introduced to the network part-way through the simulation. In all cases, it was found that the system successfully adapted and converged to the new optimal solution. *Figure 5* illustrates a simulation in which a new single new user is introduced to the test network after 1000 iterations. It can be seen that the system quickly adapts and converges to the new optimal solution.

Other experiments included removing users part-way through simulations and stochastically varying the range of each node (instead of fixing this at 56m). Simulations in which the nodes moved (instead of remaining in a fixed location) were also completed. Here, each node's motion was modeled as a random walk.

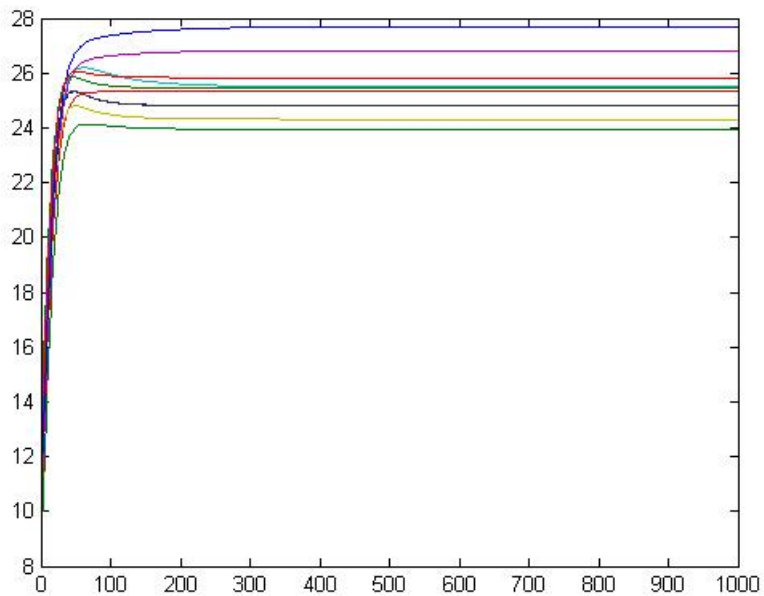


Figure 3: Plot of **utility** at each node over time.

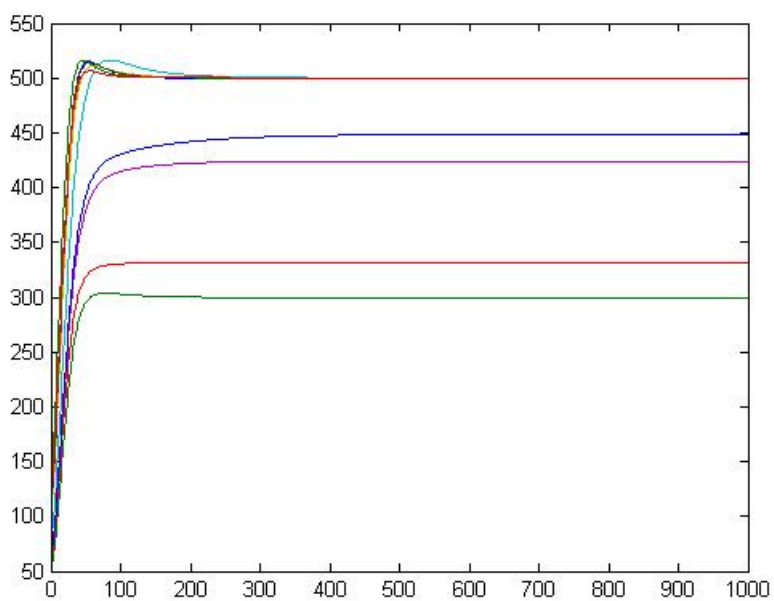


Figure 4: Plot of **power** usage $\rho_n(t)$ at each node over time.

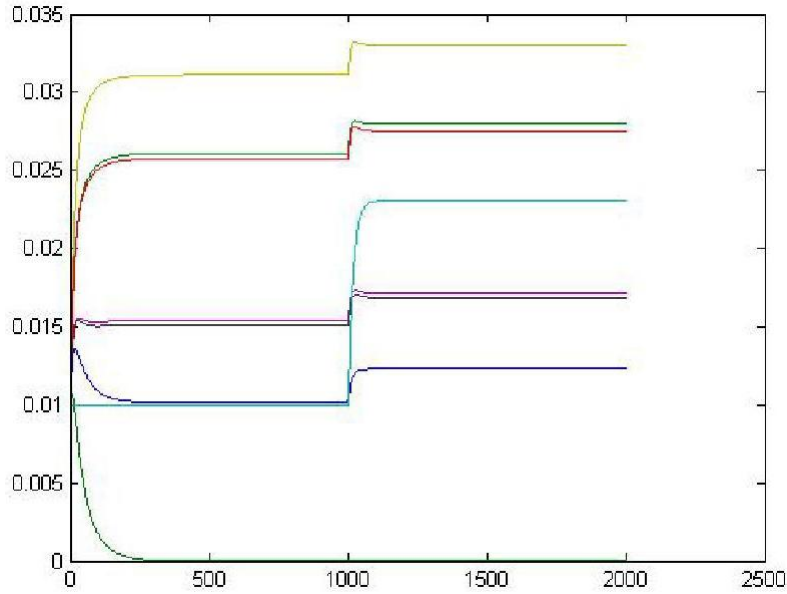


Figure 5: Plot of **prices** at each node over time.

3.6 Dynamic Solution - Utility Function Experimentation

In other simulations, the utility functions of the nodes were adjusted. Actual mobile phone networks were modeled more realistically by allowing each node to send flows to at most one other node at a time. These flows were controlled by a continuous-time Markov chain. *Figure 6* illustrates the performance of the dynamic solution under these conditions. Other utility function experiments included using utility functions that varied sinusoidally in time. Once again, the dynamic solution exhibited desirable convergence behaviour, as seen in *Figure 7*.

3.7 Dynamic Solution - Price Delay

Another aspect that should be considered when dealing with real networks is that there may be some time delay associated with information shared within the network. The success of the dynamic solution depends on each node being able to communicate its price signal to other nodes. Thus, a series of simulations were run to test the system's sensitivity to delays in the price signals. This was done by updating the flows in the network using the prices from τ previous iterations so that

$$y_r(t) = \Theta_r(\nu_s \epsilon_{s(r)}(t - \tau) + \nu_d \epsilon_{d(r)}(t - \tau) + \sum_{n \in t(r)} \nu_t \epsilon_n(t - \tau)). \quad (6)$$

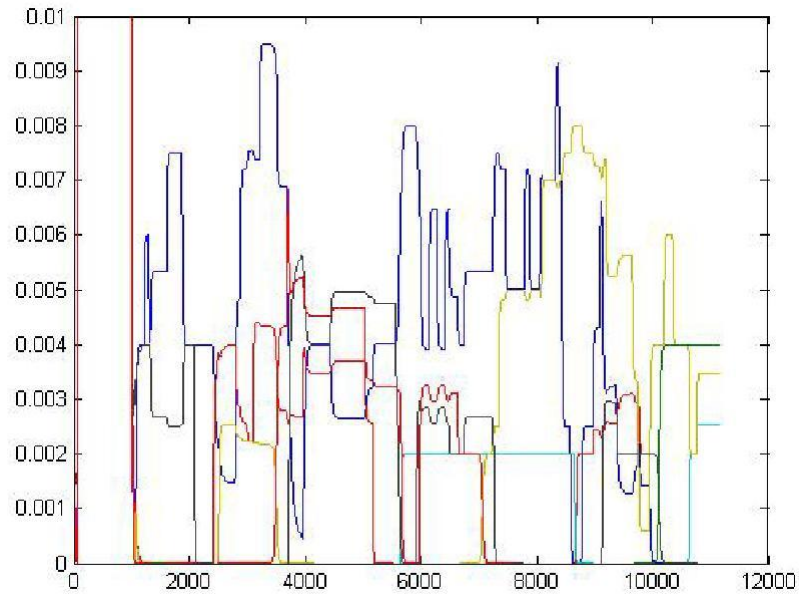


Figure 6: Plot of **prices** at each node over time.

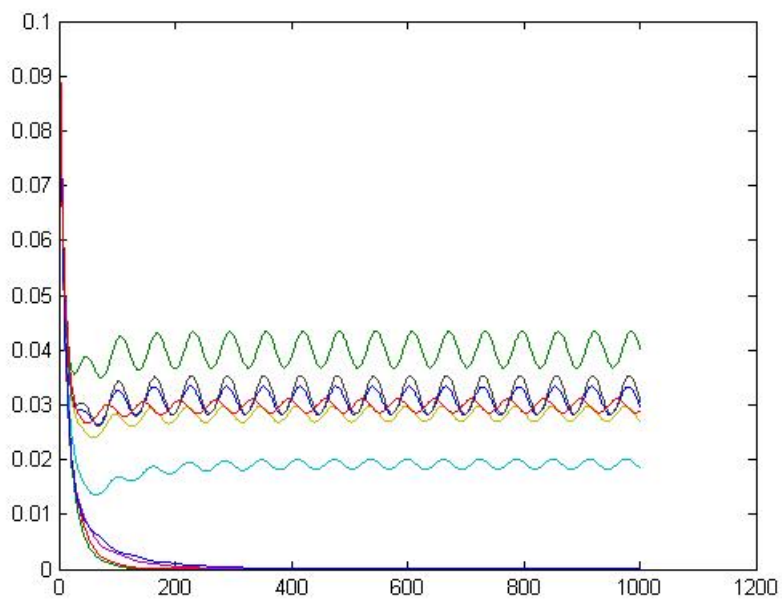


Figure 7: Plot of **prices** at each node over time.

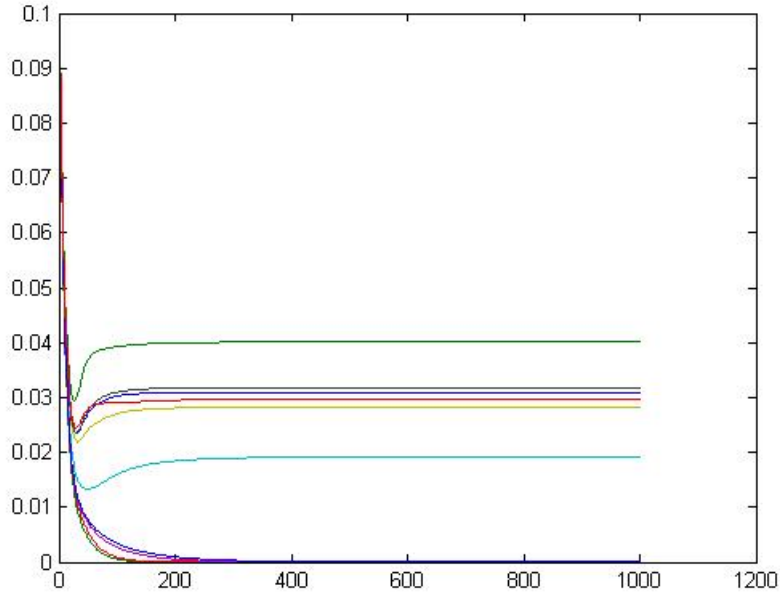


Figure 8: Plot of prices at each node over time, $\tau = 5$.

It can be seen from *Figure 8* to *Figure 14* that the dynamic solution successfully adapts provided a price delay of less than 15 iterations. However, if the delay is increased beyond this, then the system does not successfully adapt.

4 Egotistic User

The previous analysis was carried out assuming that users within the network seek to maximise total utility within the network. However, in reality it would be expected that users will seek to maximise their own utility. Thus, it would be expected that they would be unwilling to act as a transit or destination node as this requires using power for activities that do not increase their personal utility. It is therefore important to determine how egotistic users behave within the network and methods of constraining such individuals to ensure they behave in a socially optimal manner. When investigating the behaviour of egotistic users, it was assumed that all other users behave in a socially optimal manner. This may be because they are altruistic users or constrained egotists.

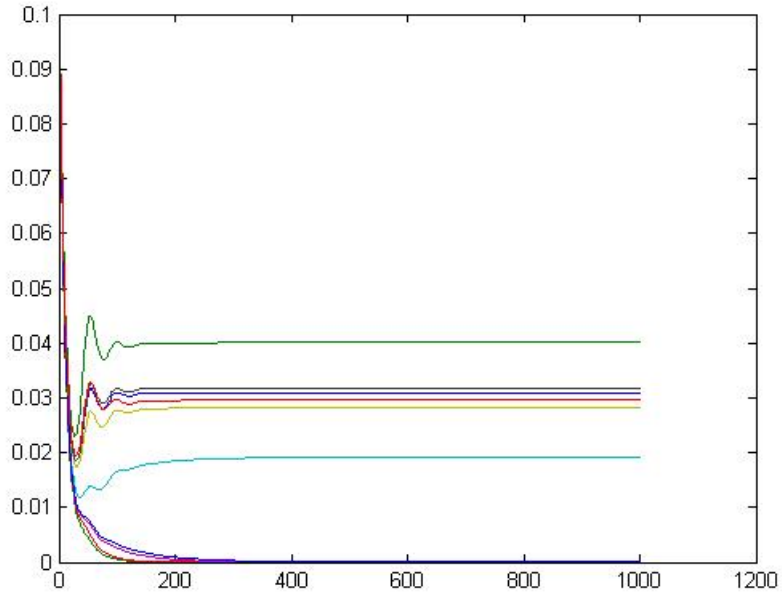


Figure 9: Plot of prices at each node over time, $\tau = 9$.

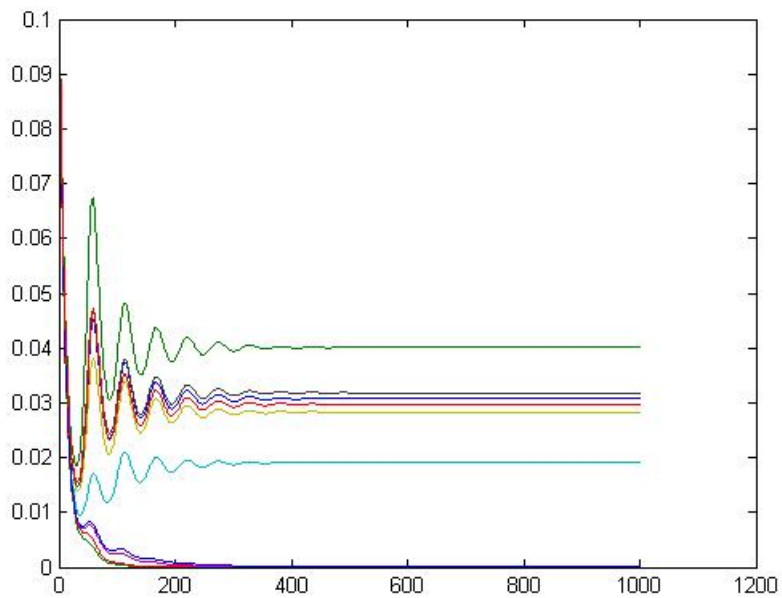


Figure 10: Plot of prices at each node over time, $\tau = 12$.

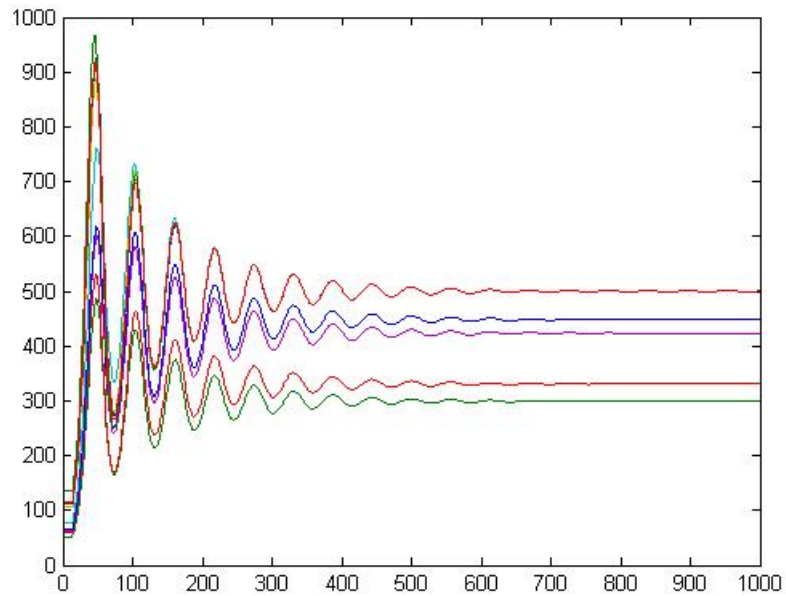


Figure 11: Plot of prices at each node over time, $\tau = 13$.

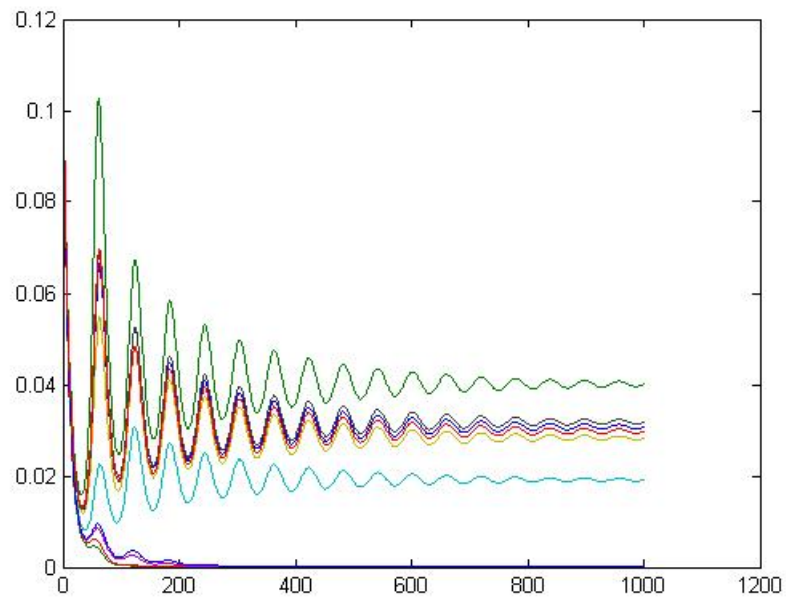


Figure 12: Plot of prices at each node over time, $\tau = 14$.

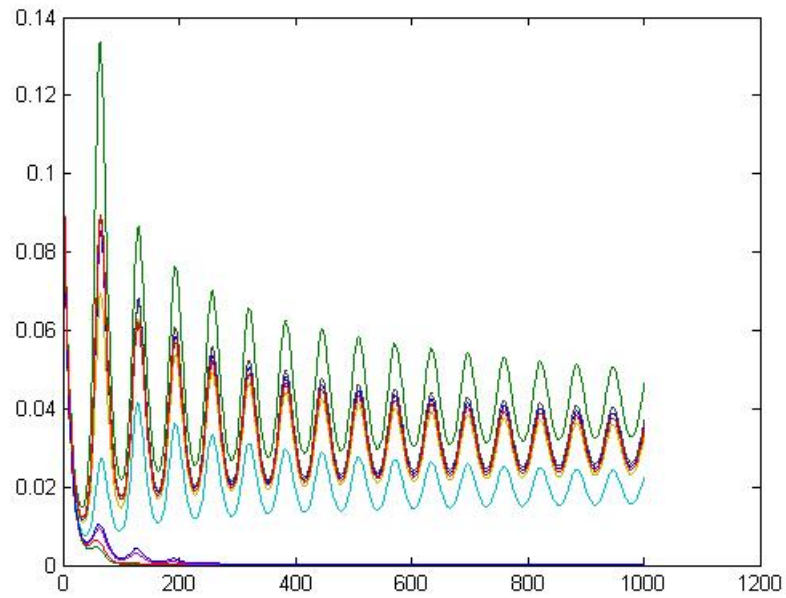


Figure 13: Plot of prices at each node over time, $\tau = 15$.

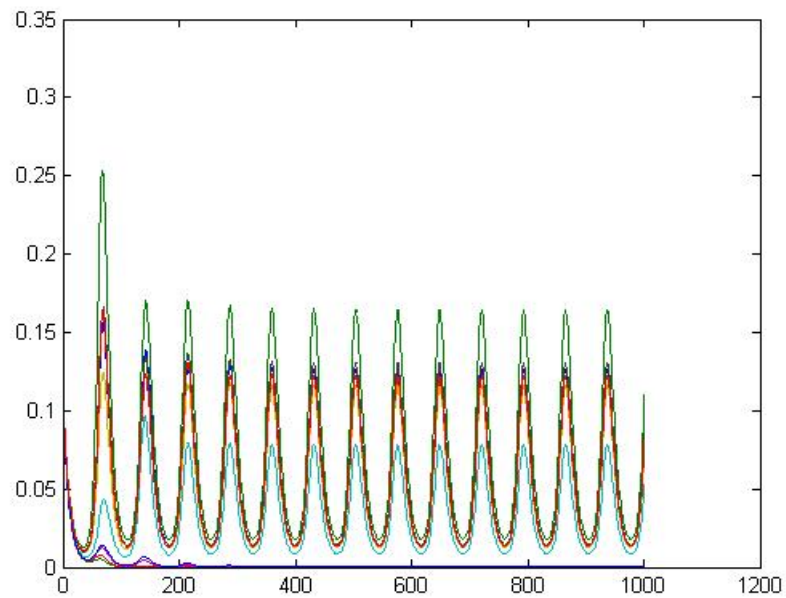


Figure 14: Plot of prices at each node over time, $\tau = 17$.

Node	Socially Optimal Utility	Egotist Utility
1	24.816	29.944
2	23.936	23.936
3	25.316	25.316
4	25.493	27.505
5	26.805	26.805
6	24.310	29.147
7	24.786	29.879
8	27.701	27.701
9	25.428	32.625
10	25.798	30.746

Figure 15: Utility achieved when each node acts as an egotist, compared to its utility in the socially optimal case.

4.1 Egotist with Global Information

It was initially assumed that if a node acts as an egotist then it cuts flows when it is a transit or destination node. The optimal flows for the egotist were then solved assuming that the egotist possesses all network information. It was assumed that other nodes are unaware of the actions of the egotist and do not adjust their own flows. Simulations in which each node in the test network becomes a egotistic user were completed.

The results of these simulations are summarised in *Figure 15*. This table compares the utility achieved by each node when it acts as an egotist to its utility in the socially optimal case. The problem with this egotist is that global information is used to compute the egotist's optimal flows. The next step is to determine if an egotistic user could achieve these utilities using only locally available information.

Note that it is not possible for the edge nodes (nodes 2,3,5 and 8) to improve upon their socially optimal utility in this network. These nodes are not utilised to full capacity in the socially optimal solution. Thus, they have no scope for gaining additional useful network space by dropping flows. This is because edge nodes are never transit nodes and are always destination nodes when receiving data. Dropping destination flows only creates additional capacity at the egotist. Since the edge nodes are all under-utilised in the socially optimal solution, this additional capacity is not useful as these nodes have nowhere to send extra data.

Node	Socially Optimal	Egotist	Symmetry	Symmetry & Adjacent
1	24.816	29.944	27.737	29.944
2	23.936	23.936	23.936	23.936
3	25.316	25.316	25.316	25.316
4	25.493	27.505	26.304	27.503
5	26.805	26.805	26.806	26.806
6	24.310	29.147	27.193	29.118
7	24.786	29.879	27.703	29.879
8	27.701	27.701	27.701	27.701
9	25.428	32.625	29.237	32.568
10	25.798	30.746	28.856	30.746

Figure 16: Utility achieved when the egotist implements the two strategies described.

4.2 Symmetry Egotist

The ‘symmetry egotist’ was the first strategy considered in which an egotistic user utilises locally available information only. This egotist simply cuts all flows in which it acts as a transit node. It then replaces this data with its own flows. The advantage of this strategy is that the prices and flows in the network are unchanged, so other users will not notice the presence of the egotist and adjust their flows.

Simulations in which each node in the test network implements this strategy were completed in *Matlab*. By referring to the ‘Symmetry’ column of the table in *Figure 16*, it can be seen that implementing this strategy resulted in an improvement upon the socially optimal utility. However, it fell short of the goal egotist utilities computed using global information. Egotist nodes could improve their utility by cutting their destination flows and then sending additional flows to nodes with excess capacity. The ‘Symmetry & Adjacent’ column of *Figure 18* shows the utility obtained if the egotist implements the ‘symmetry’ strategy and cuts destination flows and sends additional data to immediately adjacent nodes with excess capacity. It can be seen that this results in utility figures very close to the goal utility figures.

Note that the symmetry strategy is equivalent to the egotist solving their global optimisation problem, assuming all nodes within the network are utilised to full capacity. This suggests that if the egotist could determine available capacity at nodes with a price of ‘zero’ then the egotist would possess sufficient information to solve the global optimisation

problem and reach the goal utility. There are two questions that must be addressed. Firstly, how does the egotist obtain this information? Secondly, will the other nodes in the network notice the egotist and deviate from their socially optimal flows?

4.3 Egotist Tests

It is proposed that the egotist runs tests on the network to determine the available capacity at under-utilised nodes (those with a price of zero). Each of these tests have been carefully designed in an attempt to ensure that other nodes do not become aware of the egotist. It is important to note that the egotist does not actually require knowledge of the socially optimal capacities. Rather, the egotist must determine the available capacity which they could conceivably use. In some cases the egotist may not have sufficient capacity to completely determine the spare capacity of another node. However, this does not matter, because it follows that the egotist would not be able to make use of this additional, unknown capacity anyway.

The following tests, which can be implemented by the egotist, are proposed. These test will enable the egotist to determine the capacity available at this nodes that could conceivably be utilised.

- Under-utilised, adjacent nodes: The egotist cuts flows in which they are a destination node and utilises this additional capacity to increase their flows to the relevant node. The egotist continues to increase this flow until that nodes achieves a non-zero price (an increase in price). This indicates to the egotist node the excess capacity available at this node. The egotist could also use flows in which they are a transit if additional capacity is required for the test.
- Under-utilised, non-adjacent nodes: The egotist identifies the transit nodes along the route to the relevant node. They cut the flows to the transit nodes. They then send additional flows to the node of interest. For example, if the route contains a single transit node, the egotist cuts the flow to this node. They then send additional flow to the node of interest. The size of this additional flow must be half the size of the flow cut from the transit node. (Note that this assumes that ν_t is twice ν_s .)

Simulations in which the egotist implements the described tests were completed in *Matlab*. This output is shown in the ‘Egotist with Realistic Tests’ columns of *Figure 17*. It can be seen that the goal utilities computed using global information have now been achieved in a realistic manner.

Node	Egotist	Egoist with Realistic Tests
1	29.944	29.944
2	23.936	23.936
3	25.316	25.316
4	27.505	27.505
5	26.805	26.805
6	29.147	29.147
7	29.879	29.879
8	27.701	27.701
9	32.625	32.625
10	30.746	30.746

Figure 17: Utility achieved when the described tests are implemented.

4.4 Detection by Other Nodes

The other issue that must be addressed is whether other nodes within the network will be able to detect the egotist. This was done by simulating in *Matlab* what the other nodes in the network would observe if a fixed node were to become an egotist. It could then be seen if the other nodes adjust their prices and flows in response.

This was completed by doing the following:

- Running the socially optimal simulation for 1000 iterations.
- Instantaneously setting the flows along routes with the fixed node as the egotist to this node's optimal egotist flows.
- Compensate for the egotist behaviour by adding extra capacity into the network where the egotist drops flows. To elaborate, the egotist creates additional space within the network by dropping flows in which they are a transit node. The other nodes within the network are unaware of this behaviour, as the egotist compensates by replacing the dropped flows with their own data.
- The egotist artificially maintains their socially optimal price to prevent other nodes being alerted by its behaviour and adjusting their flows accordingly.

The simulations shows that the other nodes do not detect any change within the network and thus do not adjust their flows or prices. However, it is possible the other nodes may become suspicious when they stop receiving transmissions from certain nodes, which cannot be socially optimal with logarithmic utility functions.

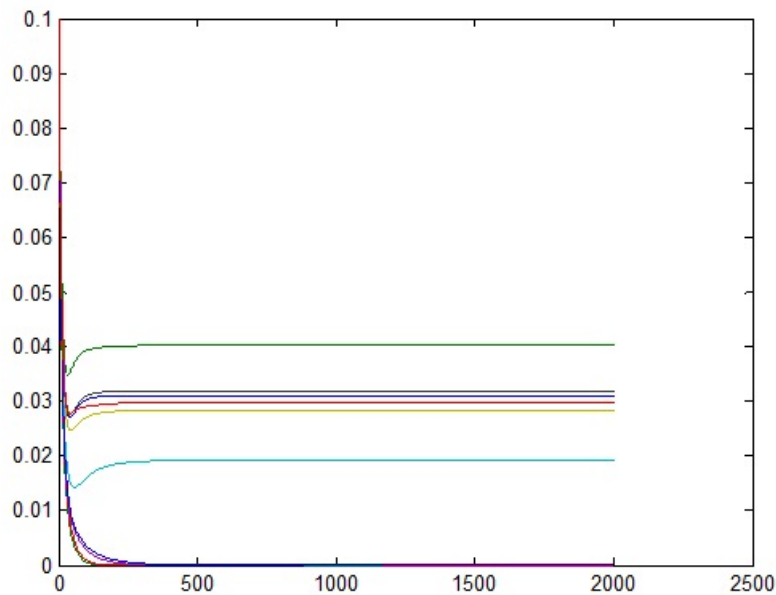


Figure 18: Plot of **prices** at each node over time.

Figure 18 and *Figure 19* shows the simulation output when node 9 is an egotist. It can be seen from *Figure 18* that the prices within the network are unchanged, which indicates that the egotist remains undetected. *Figure 19* does not show the *actual* utility received by the non-egotist nodes; it shows the utility the nodes *think* they are receiving.

4.5 Egotist with Price Manipulation

It is conceivable the egotist could manipulate their price signals in the network. If this is possible, then it would be optimal for the egotist to communicate a price of zero to the rest of the network. Thus, the egotist will attract a larger volume of traffic, which they can then cut in order to create additional space in the network for their own flows.

Simulations in which the egotist implements this strategy were completed in *Matlab*. The output is shown in *Figure 20*.

5 Further Research

Now that egotistic behaviour has been fully investigated, the next step would be to determine how to constrain users to ensure they behave in a socially optimal manner.

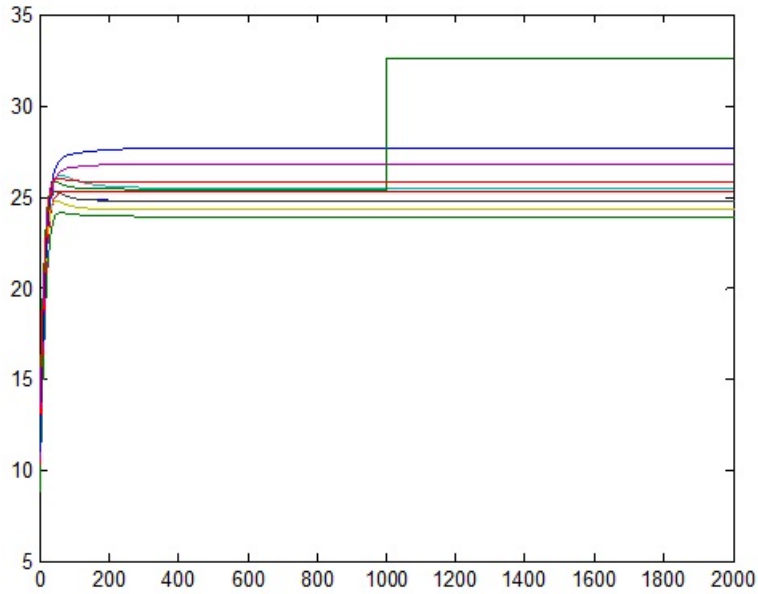


Figure 19: Plot of **utility** at each node over time.

Node	Socially Optimal	Egotist	Zero Price Egotist
1	24.816	29.944	32.738
2	23.936	23.936	23.936
3	25.316	25.316	25.316
4	25.493	27.505	29.635
5	26.805	26.805	26.805
6	24.310	29.147	31.692
7	24.786	29.879	32.921
8	27.701	27.701	27.701
9	25.428	32.625	35.786
10	25.798	30.746	34.455

Figure 20: Utility achieved when the egotist fixed a price of zero.

One possible solution is to implement some sort of credit-based system, in which users earn credits by acting as a transit node. In order to send their own data, the nodes would require sufficient credits. Nodes at the edge of the network never have the opportunity to act as a transit node, so they would require an allowance of credits.

6 Acknowledgements

This project would not have been possible without the support of my supervisors Professor Peter Taylor and Dr Mark Fackrell. All research was completed with two other students - Giles Adams and Peter Braunsteins.

I would also like to acknowledge AMSI, the CSIRO and the University of Melbourne for providing the funding which allowed this research project to be completed. I would definitely recommend the AMSI Vacation program to other undergraduate students. It was a fantastic introduction to the research side of mathematics. Attending the Big Day In was also a wonderful experience as it gave the opportunity to present mathematical research for the first time and meet other mathematics students from around Australia.

7 References

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