The University of Melbourne - AMSI Vacation Scholarship

Localised Analysis of an Ad Hoc Mobile Network

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1 Introduction

1.1 Background

Ad-hoc mobile networks are self-configuring networks of mobile handsets, connected via wireless links. All mobile handsets, or nodes, are free to move within the network and frequently change their links to one another. For such a network to function properly, it is essential that nodes are willing to act as transit nodes. That is, they forward traffic unrelated to their own personal benefit through the network. The aim of this research project was to build a simple model for these networks and solve for optimal data flows.

1.2 Model

Such a network can be modelled as a graph where vertices represent nodes and edges represent network routes. *Figure 1* shows the ten node test network used in *Matlab* simulations. This is the same network used by Crowcraft, Gibbens, Kelly and Östring in their 2004 paper. For a full explanation of the model and notation used throughout, please refer to the AMSI website for the full version of this report.

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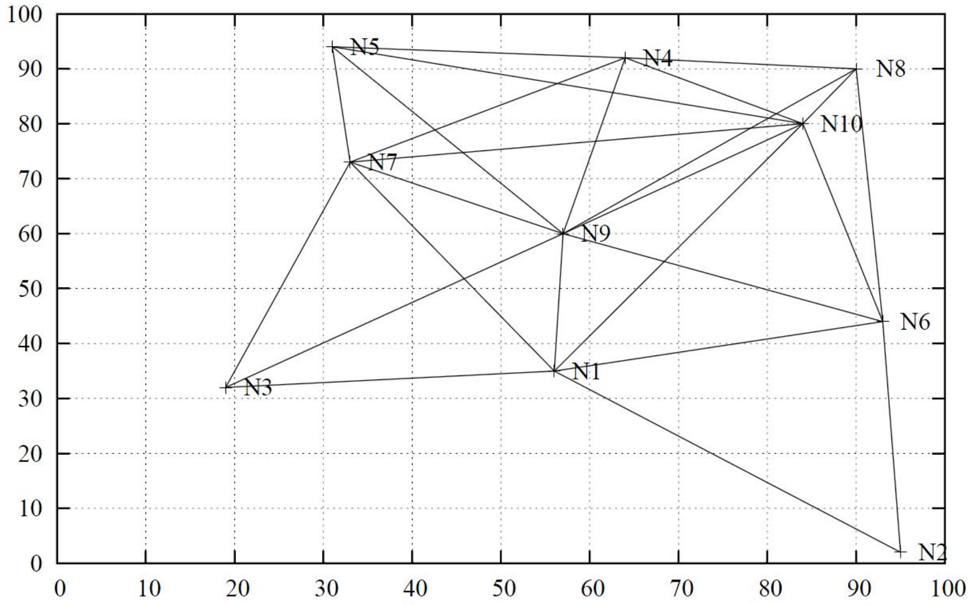

Figure 1: Our 10 node test network.

2 Static Approach

Assume that all users within the network are altruistic. That is, each user seeks to maximise total utility within the network. Krzesinski, Latouche and Taylor (2011) proposed the following static system:

$$\max_{y_r} \sum_{r \in R} \mathcal{U}_r(y_r) \tag{1}$$

subject to:

$$y_r \geq 0 \; \forall \; r \in R \tag{2}$$

$$\rho_j \leq P_j \; \forall \; j \in J \tag{3}$$

where $\rho_j = \sum_{r \in R^s(j)} \nu_s y_r + \sum_{r \in R^t(j)} \nu_t y_r + \sum_{r \in R^d(j)} \nu_d y_r$.

Note that a full explanation of all notation used can be found in the full version of this report available on the AMSI website.

This is a non-linear maximisation of a strictly concave function over a convex region. Thus, a unique solution exists. The problem can be solved utilising the Lagrange method and relevant Karush-Kuhn-Tucker (KKT) condition.

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The relevant KKTa conditions are:

$$\rho_j \le P_j$$
$$y_r \ge 0.$$

The KKTb conditions are:

$$\exists\ \epsilon_j \ge 0 \text{ such that } \epsilon_j(P_j - \rho_j) = 0$$
$$\exists\ \eta_r \ge 0 \text{ such that } \eta_r y_r = 0.$$

2.1 Static Solution

Provided $y_r \le P$ the static system has the following solution:

$$y_r(\boldsymbol{\epsilon}) = \Theta_r(\nu_s \epsilon_{s(r)} + \nu_s \epsilon_{d(r)} + \sum_{j \in t(r)} \nu_t \epsilon_j) \tag{4}$$

where $\Theta_r(\boldsymbol{\epsilon})$ is the inverse of the derivative of $\mathcal{U}_r(y_r)$.

3 Dynamic Approach

An alternative approach is to consider the network as a dynamical system in which each node emits a ‘price signal’. Other nodes use this information to update there data flows so that the system converges towards the optimal solution. A desirable property of a dynamic solution is that each node uses only locally available information (price signals and knowledge of local network topology) to update their flows. Other important properties are that the dynamical system is stable and has desirable convergence properties.

3.1 Dynamic Solution

The proposed dynamical solution is a regime in which each node updates their price on the basis of local demand

$$\frac{\partial \epsilon_j(t)}{\partial t} = \kappa \epsilon_j(t) \frac{\rho_j(t) - P}{P}. \tag{5}$$

Here, $\rho_j(t)$ is the power being consumed at node j at time t. Note that $\epsilon_j(t)$ denotes the ‘price signal’ emitted by node j at time t. Using the same function

$$y_r(t) = \Theta_r(\nu_s \epsilon_{s(r)}(t) + \nu_d \epsilon_{d(r)}(t) + \sum_{j \in t(r)} \nu_t \epsilon_n(t))$$

for flows as in the static solution, we see that as the utility function is concave, Θ_r is strictly decreasing. Therefore, a higher price at a given node implies less flows will be sent to that node. Similarly, a lower price at given node implies more flow will be sent to that node.

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3.2 Dynamic Solution - Optimality

To see that the proposed dynamic system converges to the optimal solution, start by observing the following:

- If $\rho_j(t) - P_j < 0$ then $\rho_j(t) < P_j$ and this implies that $\frac{\partial \epsilon_j(t)}{\partial t} < 0$. Thus, prices decrease.
- If $\rho_j(t) - P_j > 0$ then $\rho_j(t) > P_j$ and this implies that $\frac{\partial \epsilon_j(t)}{\partial t} > 0$. Thus, prices increase.

Intuitively, if a node is under-utilised then its price will fall until its full power capacity is utilised. Similarly, if a node is being over-utilised, then its price will rise until the power capacity restriction is met at the node.

Thus, assuming that $\lim_{t\to\infty}(\boldsymbol{y}, \boldsymbol{\epsilon}) = (\boldsymbol{y}^*, \boldsymbol{\epsilon}^*)$, it is clear that $(\boldsymbol{y}^*, \boldsymbol{\epsilon}^*)$ are optimal flows and prices respectively.

3.3 Dynamic System - KKT Conditions

It is also clear that $(\boldsymbol{y}^*, \boldsymbol{\epsilon}^*)$ satisfy the KKT conditions. To see this, recall

$$\frac{\partial \epsilon_j(t)}{\partial t} = \kappa \epsilon_j(t) \frac{\rho_j(t) - P}{P}$$

and note that for stable prices ($\frac{\partial \epsilon_j(t)}{\partial t} = 0$), it is required that $\epsilon_j(t) = 0$ and/or $\rho_j(t) = P$. Thus, the necessary KKT conditions are satisfied and the socially optimal solution is a fixed point of the proposed dynamical system. Further, the proposed dynamical solution is a system with a single fixed point (by construction).

3.4 Dynamical System - *Matlab* Simulations

In order to confirm that the proposed dynamical system behaved as predicted, *Matlab* simulations were conducted. These used the simple test network (see *Figure 1*), a discretised version of the differential equation and simulations ran until the system converged. The numerical solution computed using the dynamic system was then compared to results computed using the static solution by Krzesinski, Latouche and Taylor (2011). It was found that the numerical results for the dynamical solution matched those obtained by Krzesinski, Latouche and Taylor (2011) for the static solution. From *Figures 2* it can be seen that the dynamic solution displayed desirable convergence properties.

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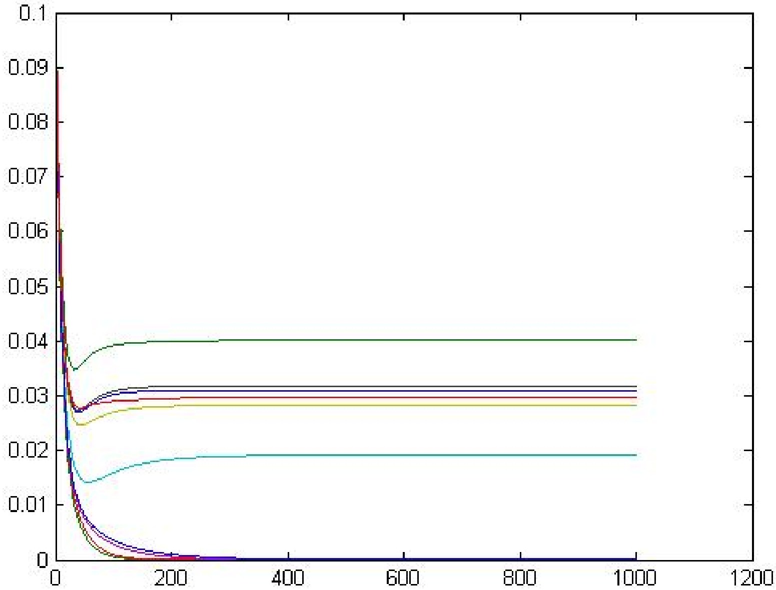

Figure 2: Plot of **prices** at each node over time (simulation iterations).

4 Acknowledgements

I would like to acknowledge AMSI, the CSIRO and the University of Melbourne for providing all funding for this research project. I would recommend the AMSI Vacation program to other undergraduate students. It was a fantastic introduction to the research side of mathematics. Attending the Big Day In was also a wonderful experience as it gave the opportunity to present mathematical research for the first time and meet other mathematics students from around Australia.

5 References

For the full version of this report, please refer to the AMSI website.

Krzesinski, A.E., Latouche, G. and Taylor, P.G. 2011, How do we encourage an egotist to act socially in an ad hoc mobile network?, paper presented to the Applied Probability Reading Group, University of Melbourne, 12 December 2011.

Crowcraft, J., Gibbens, R., Kelly, F. and Östring, S. 2004, Modelling Incentives for Collaboration in Mobile Ad Hoc Networks. *Performance Evaluation*, 57 (2004), 427-439.