Melting crystals in microgravity

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This project was concerned with results of the Isothermal Dendritic Growth Experiment, which was conducted on the space shuttle *Columbia*. In these experiments, crystals were formed by supercooling pivalic acid, before being melted by a surrounding heat source. Data collected on video frames suggests that a crystal generally melts in the shape of a prolate spheroid. The aspect ratio of the spheroid increases initially, and then shrinks as the crystal melts completely. My project was to model the melting of a single dendritic fragment, and to use this model to describe the results in the experiments.

The mathematical model employed took form of a moving boundary problem, with the governing equation (Laplace's equation) holding in exterior of the dendrite. For an initial shape of the dendrite, the mathematical problem was to track the evolution of the dendrite as it melted. The governing equation was derived from the heat equation under the physically justified assumption that the melting process was slow. We first considered an infinite-domain problem, for which asymptotic analysis was possible by applying a Baiocchi transform. We ascertained that the aspect ratio of a melting dendrite remains constant in this case, contrary to the experiments.

Next, a finite-domain problem was investigated. Its complexity necessitated the use of a numerical scheme. By solving the moving boundary problem in MATLAB using the finite element method, it was found that the aspect ratio of the dendrite increases monotonically as the crystal melts. This was consistent with the first part of the experimental results.

In order to replicate the full experimental results, surface tension was incorporated on the crystal interface. This resulted in the aspect ratio increasing gradually, before eventually decreasing (since surface tension acts to smooth out regions of high curvature). While previous authors had proffered that surface tension is indeed the cause for this phenomenon, this had never before been confirmed by solving a full moving boundary problem. My project demonstrated that, with appropriate parameter values and dimensional scalings, the results of the Isothermal Dendritic Growth Experiment can be reproduced by including a curvature-driven boundary condition.

This project gave me invaluable research experience and introduced me to hitherto unfamiliar mathematical concepts. In particular, it encompassed a broad range of asymptotic and numerical techniques. I sincerely thank AMSI for their generous support of this project and the opportunity it provided. Thanks also to my wonderful supervisors Scott McCue and Tim Moroney.

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