Sheaves and Schemes

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One of the joys of studying mathematics is learning about unexpected category equivalences.

A category is a collection of objects with, for every pair of objects, a set of what we call morphisms between the objects. These morphisms must obey the condition that if we have any three objects, and morphisms from the first to second, and second to third, objects, then we have a morphism from the first to third objects. More intuitively, though, a category can simply be thought of as a collection of points connected by a collection of arrows. This is an incredibly general construction, and many classes of objects studied in mathematics can be considered categories in a natural way. An example of a category is that of sets and functions.

Two categories are equivalent, then, if we may take one category and, merely relabelling the objects and morphisms, end up with the other category. This must respect the composition structure of the morphisms. That is, if morphisms A and B compose to the morphism C in the first category, then the relabellings of A and B compose to the relabelling of C in the second category. What this says is that the objects in the two categories correspond to each other, and furthermore the relationships between objects are the same. This is very powerful.

For example, a question about one category can often be asked in an equivalent category, where it may be much easier to answer. Such a process is at the heart of much modern mathematics, and is the key behind say, the proof of the insolubility of the quintic, a question that lay asked but unanswered for many centuries.

I spent the last summer learning about a category equivalence known as GAGA. Proved in the 1950s by Jean-Pierre Serre, it states an equivalence between certain categories of algebraic and analytic spaces. This is fascinating and surprising, as it

thus details a deep relationship between algebraic and analytic geometry. It also opens up the use analytic techniques in attacking algebraic problems, and vice versa, and consequently has many and varied applications.

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