The Magnetic Prandtl Number and Dynamo Theory

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My summer project explored an area of Magnetohydrodynamics (MHD) known as Dynamo Theory. Specifically, bodies such as the sun are composed of conducting fluid which responds to magnetism. There is thus a coupling of two vector fields – the velocity field of the fluid and the magnetic field. It is possible to derive from Maxwell's Equations, Ohm's Law in a Moving Medium, the Navier-Stokes Equation and the Lorenz Force Law two equations known as the Induction Equation and the Momentum Equation. These are coupled PDEs describing the time evolution of the magnetic and velocity fields respectively. Such a system, which maintains its magnetic energy by resisting dissipative effects over long time scales, is referred to as a dynamo.

The challenge of the project was to solve these equations for given sets of initial conditions and parameters. In order to minimise error, the equations were transformed into Fourier space. In Fourier space, differentiation is an eigenfunction with the Fourier wave numbers as eigenvalues. This means that vector calculus operations become trivial arithmetic in Fourier space. The resultant equations were then timestepped using a Semi-Exact Adams Bashforth scheme. This uses a trick somewhat similar to an Integrating Factor to again keep the error as small as possible.

The results of the project were highly intriguing. Firstly, previously obtained results from earlier academic work performed by my supervisor were replicated. The thrust of the project, however, was to investigate the effect of the Magnetic Prandtl Number. This is the ratio of the kinetic and magnetic diffusivities. It can be proved that there exists a boundary which, when crossed, cannot support a dynamo - that is, the kinetic energy goes to zero.

My simulations confirmed this result. Starting from an initial stable dynamo the Prandtl Number was varied. It was observed that below a certain value a dynamo exists and above the value the dynamo collapses. Moreover, when using values for which the dynamo exists it appeared that the stable solutions exhibited decreasingly aligned magnetic and velocity fields. Indeed, the crosshelicity (a normalised measure of vector field alignment) appears to have an equilibrium value dependent on the Prandtl number. Results indicate that the equilibrium crosshelicity is exactly a quadratic function of the Prandtl number. This would seem to be a deep theoretical insight which would be worthy of further exploration.

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