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Geometric Topology – The Classification of Compact 2-Manifolds

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In the summer of 2005/2006 I was the happy recipient of an AMSI vacational scholarship. I had enjoyed an excellent introductory point set Topology course in my third year of study at Latrobe University and chose to further explore some topics in Geometric (or Classical) Topology under the supervision of Dr John Banks.

A Compact 2-Manifold is a closed and bounded set that is locally homeomorphic to Euclidean 2-space (ie – it's a surface). The Classification Theorem says that all surfaces are Topologically equivalent to either a sphere, a connected sum of n Tori (think of a doghnut with n holes), or a connected sum of m projective planes (think of a sphere with m discs cut out and m möbius bands glued on to the boundaries – the key feature is non-orientability).

The Classification Theorem was the general focus of my project. I worked through the standard proof and presented it to members of the maths department. The proof relies on a result due to Rado (1925) that states that any compact 2-manifold can be triangulated (chopped up into triangles that only meet in single vertices, complete edges, or not at all). This is a good example of an 'obvious' result in Topology that is anything but obvious in proof (cf. Jordan's seperation theorem). It is 'hand waved' in most standard texts which give a proof of the Classification Theorem (including the celebrated ZIP proof due to John Conway). The main research component of my project consisited of understanding and presenting this proof.

I would highly recommend an AMSI summer scholarship to any serious undergraduate maths student. The experience I gained in research, presentation, and supervised self study is invaluable. Thanks to AMSI, Latrobe and my supervisor.

Rohan received an ICE-EM Vacation Scholarship in December 2005.
See www.ice-em.org.au/students.html#scholarships05