INTERNATIONAL CENTRE OF EXCELLENCE FOR EDUCATION IN MATHEMATICS

Investigation of cyclotomic polynomials

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During the summer vacation 2004-2005, I was the recipient of an AMSI scholarship. The project I undertook involved investigation of cyclotomic polynomials, and was supervised by Dr Grant Cairns.

Cyclotomic polynomials are polynomials whose roots are the primitive n^{th} roots of unity. These polynomials have no non-trivial factorisation with rational coefficients. The n^{th} polynomial, $\Phi_n(x)$, has degree $\phi(n)$ (i.e. the Euler phi function), and can be calculated inductively.

In the summer project, I investigated the coefficients of these polynomials. In particular, we were interested in the value of the middle coefficient, and the values of n for which this coefficient was not equal to 0,1 or -1. We then considered the value of the sum and the alternating sum of the coefficients of the n^{th} cyclotomic polynomial. This idea was then extended by evaluating the cyclotomic polynomials at i. The value of the real and imaginary coefficients would therefore be the alternating sums of the coefficients of even powers of x and of odd powers of x respectively. We then formulated conjectures giving formulae for these values.

The AMSI scholarship was a good opportunity to obtain first-hand experience of research, and to learn about topics that I otherwise would not have studied. It also provided a chance to gain experience using various maths software packages. I highly recommend undertaking an AMSI vacation scholarship to any keen maths students.

Sarah received an ICE-EM Vacation Scholarship in January 2005.
See www.ice-em.org.au/students.html#scholarship.