The inverse problem in the calculus of variations: non-existence of Lagrangians

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For the two dimensional case, to find a Lagrangian from a pair of forces, we apply a process which determines the coefficients of a 2x2 symmetric matrix and then we check whether the matrix is invertible. If it is, then a Lagrangian exists. If it is not invertible, that is, it has a zero determinant, then a Lagrangian can't be found. The project was about finding the necessary conditions on the forces such that the matrix will not be invertible. We used the exterior differential systems approach to find these conditions.

$$\begin{pmatrix} \alpha & 0 \\ 0 & 0 \end{pmatrix} \quad \begin{pmatrix} \alpha & \alpha c \\ \alpha c & \alpha c^2 \end{pmatrix}$$

Here are two symmetric matrices that have a zero determinant. The arguments in the matrices are functions and not necessarily numbers. The first one was already completed by my supervisor, Geoff Prince. The second matrix was one that I completed. We could find the conditions required for the forces to produce the second matrix; however we could not find any examples for this so it is possible that it can't occur. Another thing that we did was to check whether or not we could transform the second matrix into the first matrix by a change of coordinates. It turns out that we could not find a way to do this. If we had found a way of doing this then the second matrix would not have needed to be done.

The project was fun; it was quite different from the way mathematics is taught ordinarily. I think it gives you an idea about what an honours project is like. I found it generally rewarding but frustrating at times.

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