INTERNATIONAL CENTRE
OF EXCELLENCE FOR
EDUCATION IN
MATHEMATICS

Minimal Submanifolds

David Hartley, School of Mathematical Sciences, Monash University

The aim of this project was to investigate isometric immersions of 2-mainfolds into R3 and determine when such an immersion exists. The 2-manifolds were represented using quaternion valued spinors. The relationship between the immersion, F, and the spinor, ψ, is given by

$$dF = \alpha = \overline{\psi}\beta\psi$$

Where β is a given imaginary valued quaternion 1-form and when the spinor is restricted to unit length its magnitude squared is the metric on the image. As the spinor is related to the derivative of the immersion it is necessary to know the integrability condition on the spinor. F exists locally if and only if dα = 0 or in terms of spinors

$$\beta \wedge d(\psi)\overline{\psi} = p + d\beta / 2$$

Where p is any real function. An isometric immersion is one that preserves distances between points; hence its induced metric is the same as the metric on the manifold. The problem of determining when such an immersion exists was tackled by assuming an immersion F_0, with corresponding ψ_0 and β_0, such that its induced metric is close to the required one. By using the variations $\beta = \beta_0 + \varepsilon\beta_1$ and $\psi = \psi_0 e^\lambda$ so that $|\beta|^2$ is the required metric and $\lambda = \varepsilon\lambda_1 + \varepsilon^2\lambda_2 + \ldots$. As the change is small only λ_1 needs to be determined, which is done by the following equation

$$d(2dF_0 \times \lambda_1 + \overline{\psi}_0 \beta_1 \psi_0) = 0$$

While it was not found under what circumstances this equation has solutions this equation can be used to determine the spinor necessary for an isometric immersion. This vacation scholarship introduced me to many new mathematical ideas and enhanced my learning greatly in preparation for a year of honours. The Big Day In was also a very valuable tool in learning about other areas of research that can be undertaken.

David received an ICE-EM Vacation Scholarship in December 2006.
See http://www.ice-em.org.au/students.html#scholarships2007