INTERNATIONAL CENTRE
OF EXCELLENCE FOR
EDUCATION IN
MATHEMATICS

General Investigations into the structure of 3+1 dimensional Space Times
Martin Koslov, School of Mathematics, Monash University

My summer vacation project was conducted under the supervision of Professor Robert Bartnik. The aim of the project was to investigate certain spaces prescribed by a specific metric, in particular to investigate the properties of such spaces via the Einstein field equations described below:

$$G_{ab} = 8\pi T_{ab}$$

For the most part, the summer was spent learning the necessary mathematical tools required to tackle the problem, namely differential geometry. Curvature became the central focus for the first couple of weeks as it was crucial in understanding the field equations described above.

Various techniques of curvature were investigated, especially the method of moving frames by Ellie Cartan. Components of the Riemann curvature tensor were calculated using the following equations:

$$\Omega_{ij} = d\omega_{ij} + \omega_{ik} \wedge \omega^{k}{}_{j} = -R_{ijkl} e^{k} \wedge e^{l}$$

Once differential geometry was learnt to a sufficient degree, the focus of the project shifted to investigating peculiar spaces. One such example is the Friedmann universe, which describes a homogeneous and isotropic space time modelled by the following metric:

$$ds^2 = -dt^2 + R^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2(\theta) d\varphi^2\right)$$

In conclusion, the vacation project provided further insight into mathematical structures not normally encountered in a standard undergraduate degree. The Big Day In also provided an opportunity to learn about the different branches of mathematics on offer.

Martin received an ICE-EM Vacation Scholarship in December 2006.
See http://www.ice-em.org.au/students.html#scholarships2007