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Mutually Orthogonal Latin Squares

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For 6 weeks during the 2006/2007 summer break I undertook a vacation scholarship with Dr Ian Wanless at Monash University. The topic I studied was Mutually Orthogonal Latin Squares (MOLS) and the aim was to find some evidence of a lower bound on the number of MOLS with respect to the order of the squares.

Latin squares of order n are n by n matrices whose entries are elements of some set of n unique symbols. The elements of a latin square are arranged such that each row and column are a permutation of this set of symbols. MOLS is the term for a collection of latin squares in which each individual square is said to make an orthogonal pair with all other squares in the collection.

Latin squares of prime power order p are known to have a set of MOLS of size p-1. This is the upper bound for all orders. Therefore we set out to find a lower bound for non-prime power orders, we looked at orders n = p+1 for some prime p.

The number of Latin squares has super exponential growth with increasing order, therefore we used a subset of latin squares and constructed algorithms to compute the size of MOLS for a given subset. The trick was to find a small enough subset that still produced a lower bound. The first subset used was diagonally cyclic latin squares (Wanless 2003) but this only allowed computation up to a small order (around n=13). The second subset was generated by applying a number of quadratic orthomorphisms. This allowed large computation but failed to find a stable lower bound. This has however given further insight to the behaviour of subsets of latin squares and will assist with future studies.

The AMSI summer scholarship is a great opportunity to gain an insight into new areas in mathematics and meet people with expertise in a broad range of fields in the mathematical community. It has been an invaluable experience and will no doubt assist me in my future career decisions and academic endeavours.

James received an ICE-EM Vacation Scholarship in December 2006.
See http://www.ice-em.org.au/students.html#scholarships2007