INTERNATIONAL CENTRE
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EDUCATION IN
MATHEMATICS

Function spaces and interpolation of operators

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The aim of my project was to study the basics of several function spaces. These are the Lebesgue spaces $L^P(R^n)$, the Sobolev spaces and the Lipschitz spaces. From this I also had a brief introduction to the estimates of the boundedness of singular integrals on L^P spaces.

For this project I relied on two main texts. The first was *Singular Integrals and Differentiability Properties of Functions* by Elias M. Stein. In this text, the chapters of focus were Chapters 1, 2 and 5. The other text was *Fourier Analysis* by Javier Duoandikoetxea and the chapters included 1, 2, 3 and parts of 4.

Topics that I studied included the

- Classical Marcinkiewicz Interpolation theorem for L^P boundedness of sublinear operators
- important Calderón-Zygmund decomposition
- well known Hörmander condition of an L^1 function for weak type (1, 1) estimates of an operator which is bounded on L^2.

Boundedness of operators comes in two forms, strong type and weak type. Strong type implies weak type but not the other way around. The interpolation theorem of Marcinkiewicz (classical) says that if an operator is bounded weakly (p, p) and also bounded weakly (q, q), then it is bounded strongly on $L^r(R^n)$ for $p < r < q$. Some time was spent understanding this theorem and how it fit into the larger context of Calderón-Zygmund theory on singular integrals.

Some examples of the applications of these results include

- L^P boundedness of the Hilbert transform on the real line
- Riesz transform on R^n
- more general operators whose kernels satisfy the so-called Calderón-Zygmund conditions

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