K-Theory and D-Branes

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String Theory is a branch of theoretical physics which attempts to describe elementary particles within atoms in terms of one-dimensional oscillating lines. Describing the dynamics of open strings requires specifying boundary conditions for the endpoints of the string. In string theory, there are two types of allowable boundary conditions for the endpoints: Neumann and Dirichlet boundary conditions. Dirichlet boundary conditions fix the endpoints of the string so that they are constrained to move on a specified object, which we call a D-Brane.

If X is the spacetime manifold, then a sub-manifold Q is a D-Brane if, for a given string worldsheet Σ mapped to X, we have $\partial\Sigma$ mapped to Q. A simple calculation shows that in the presence of Dirichlet boundary conditions (and hence D-Branes), the momentum of the string is not conserved. For a long time string theorists did not consider Dirichlet boundary conditions to be possible in the context of string theory for this reason. It turns out that it is possible for strings to be attached to D-Branes, and furthermore, string momentum is conserved − the momentum lost by the string is transferred to the D-Brane. This idea began the investigation into D-Branes as dynamical objects, and remains an area of active research today.

D-Branes are classified by the (spatial) dimension of the D-Brane worldvolume. A D_p-Brane is a p-dimensional manifold, with a (p+1)-dimensional worldvolume. Since spacetime has 10 dimensions in Superstring theories, a string attached to a D_9-Brane has endpoints that can move throughout all of space.

In string theory, the different vibrational modes of the strings give rise to different elementary particles. It is therefore clear that the presence of D-Branes influences the spectrum of particles obtained from the theory. Consider, for example, two parallel D_2-Branes with a string stretched between them. Strings have a natural tension which tends to shrink the string, thus work must be done in order to stretch the string. Stretching the string increases the energy, and hence mass of the string. A string stretched between two D_2-Branes therefore has a minimum length and mass, determined by the distance between the D-Branes.

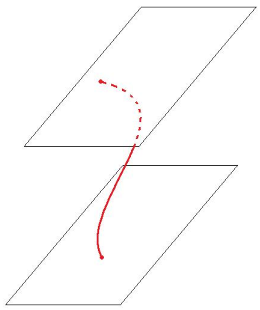

Figure 1: A string stretched between two parallel D_2-Branes

To motivate our discussion of D-Branes in the context of K-Theory, we consider an arrangement of n D_p-Branes which are coincident - that is, they all lie on top of one another. Strings are able to stretch from one D_p-Brane to another, and it turns out that the zero mass states in the spectrum of this system[1] are a set of interacting quantum fields which give rise to a $U(n)$ gauge theory. For a single D_p-Brane, the massless particles are photons. They obey p-dimensional analogues of Maxwell's equations of electromagnetism, so that even singular D-Branes carry an electromagnetic field on their volume.

In Type IIB string theory, Sen's conjecture implies[2] that D_9-Branes and anti-D_9-Branes can annihilate by a process known as Tachyon Condensation. Furthermore, that every possible configuration of D-Branes can be obtained by this process. We label a coincident configuration of n D_9-Branes and m anti-D_9-Branes by the pair, (E, F), where E is a $U(n)$ complex gauge bundle carried by the D-Branes and F is a $U(m)$

[1] At least for low energies

[2] This argument assumes that there is no non-trivial background B field. It has been argued that D-Brane charge in the presence of a non-trivial background field is classified by Twisted K-Theory

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complex gauge bundle carried by the anti-D-Branes. The invariance of the D-Brane charge under Tachyon condensation leads us to regard as equivalent the configurations (E, F) and $(E \oplus H, F \oplus H)$, since the configuration (H, H) can annihilate to the vacuum.

The set of allowed configurations, and hence the charge of the D-Branes, is therefore classified by pairs of complex vector bundles over the spacetime X, together with the above equivalence relation. This is the very definition of the K-Theory of a topological space X, and we conclude that D-Brane charge is classified by the K-Theory of Spacetime.

Although this argument is based on Tachyon condensation, a process which is not well understood in either mathematics or physics, the conclusion is well supported by other arguments. In particular, Witten argues that anomaly cancellation in the presence of D-Branes requires that the normal bundle of the D-Brane worldvolume, $\nu(Q)$, carries a $\mathrm{Spin}^{\mathbb{C}}$ structure. This condition implies that the third integral Stiefel-Whitney class, $W_3(\nu)$, vanishes, a condition required in order to define the charge as an element of K-Theory.

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