Reflection Groups

Sam Chow
University of Melbourne

A reflection is an *n*x*n* invertible matrix with complex entries. A reflection group is a finite group generated by reflections.

A classification of all irreducible reflection groups [1] reveals that are 34 primitive irreducible reflection groups. Cohen [2] provided a starting point – specific data for those of rank 2. And so we began what would be a painful yet rewarding week of algebra: learning definitions, explicitly calculating matrices and subgroups, constructing diagrams...it was well worth the effort.

A 2008 result demonstrates a one-to-one correspondence between reflection groups connected p-compact groups [3]; ultimately, it is hoped that study into reflection groups will help mathematicians to understand p-compact groups. Analogous to the Inductive Polynomial Realization Theorem [3], we wanted to prove that primitive reflection groups were determined by their proper subspace stabilisers; we achieved this for the rank 2 irreducible case.

A theorem by Chevalley, Shephard and Todd [4] then led us to study rings of polynomial invariants, proving analogous results between these and reflection groups, and vice versa. Finally, we embarked on further study of Shephard groups, and aim to write a short paper on our findings.

The project was fascinating and rewarding. I would like to thank the following:

- AMSI
- Department of Mathematics and Statistics, University of Melbourne
- Professor Arun Ram, University of Melbourne (my supervisor)
- CSIRO
- Quynh Chi Nguyen and Natalie Aisbett for their input

Sam received an AMSI Vacation Scholarship in December 2008
See: www.ice-em.org.au/vs09.html

References

[1] Shephard G. C., Todd J. A., "Finite unitary reflection groups", *Canadian J. Math. 6* (1954): 274–304

[2] Cohen, A. M. “Finite complex reflection groups”, Annales scientifiques de l'École Normale Supérieure, Sér. 4, 9 no. 3 (1976): 395

[3] Andersen K. K. S., Grodal J., Møller J. M., Viruel A. The classification of p-compact groups for p odd. Ann. of Math. (2) (2008) 167(1):95–210.

[4]Chevalley, Claude (1955), "Invariants of finite groups generated by reflections", American Journal of Mathematics 67: 778–782. See also note 1.

Sam received an AMSI Vacation Scholarship in December 2008
See: www.ice-em.org.au/vs09.html