Mathematical Biology: Cells on a Growing Tissue

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My project was to investigate how cells behave on a growing tissue. In the model studied, cells are allowed to proliferate logistically and move diffusively, but are also swept downstream by advection as the tissue grows. This model can be used to study the migration of nerve cell precursors down the gut in the developing embryo. Occasionally, colonisation does not occur fully and the nervous system develops incorrectly (Hirschsprung's disease). Hence, one of the main things that I focused on was the conditions required for proper colonisation.

The PDE used to model the cell density is second order and nonlinear and cannot be solved exactly. I studied some simplified problems analytically and numerically to get an idea of how cells behave in this environment.

To proceed with the analysis, approximations were made to obtain a linear PDE. First, the domain was assumed to grow exponentially, leading to simple advection. Second, the cells were also assumed to grow exponentially, an approximation that holds well for low cell densities. Finally, the boundary conditions were ignored and the problem solved by separation of variables.

When considering colonisation, the most important thing is the position of the wavefront relative to the domain boundary. A threshold density at the wavefront is tracked. This is plotted on a spacetime diagram and compared to the movement of the boundary; if they intersect, this is identified with colonisation of the domain.

I found that intersection always occurs if the cell growth rate is faster than the advection speed. Density may fall below the threshold, but always recovers after some time. If advection dominates, cell density always falls below the threshold level, but intersection may occur first, depending on the initial conditions and the values of certain parameters.

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See: www.ice-em.org.au/vs09.html

For the numerical work, I started by successfully modeling the diffusion equation using a one-dimensional random walk. Inclusion of a logistic growth term also approximated the behaviour of the expected solutions. Unfortunately, introducing advection proved difficult because the model only converges to the solution of the desired PDE for small advection speeds.

This was an interesting project and gave me a taste of research in mathematical biology. I also learnt how to use some useful programmes such as Mathematica, Matlab and LaTeX.

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