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Mathematical Table Turning

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"Put a square table on an irregular surface and chances are it will wobble. However, by just turning it on the spot, you can always find a position in which all four legs touch the ground." Burkard Polster and Marty Ross(2005)

The above "Picnic Table Problem" was the starting point for this project. The aim of the project I undertook was to review papers on this problem with the objective to understand, summarise and build on their contents. The three papers I focused on were Baritrompa, Löwen, Polster, Ross; Fenn and Meyerson. I gave a presentation of my understanding of the problem at the CSIRO conference in February. As it had been some time since last studying, understanding the papers and filling in details on proofs became the focus of the project with the view to extend the problem into my honours year.

The "Picnic Table Theorem" has appeared in a few journals notably in *Scientific American* in a column by Martin Gardner(1973) and in *Vinculum* in a column by Burkard Polster and Marty Ross(2005). These appearances were accompanied by intuitive explanations using the I.V.T. However it is easy to think of discontinuous grounds for which this will not work, so for a more rigorous proof and conditions for the ground for which this is true one must delve a little deeper such as in the papers by Baritrompa et al., Fenn and Meyerson.

To begin to look at the problem mathematically we must first attempt to reduce the problem to mathematical ideas. In Baritrompa et al., two different tables are described: a "mathematical table" and a "real table". The mathematical table consists of a square whose centre is where the diagonals meet. The mathematical table is considered to be "locally balanced" if all four vertices touch the ground and the centre of the table is on the z-axis. A real table consists of a solid square top and four legs of equal length at right angles to the top. The endpoints of the four legs form the vertices of an associated mathematical table and the real table is considered to be locally balanced if its associated mathematical table is locally balanced and no part of the table top or legs are below the ground.

In Baritrompa et al., a theorem by Livesay is used to prove the following theorem.
Theorem: A mathematical table can always be balanced locally, as long as the ground function is continuous.

This gives us an existence of the balance point for mathematical tables however it gives no means of finding such a position or a means to restrict the function for real tables. Restriction of the function is required as it is easy to think of continuous grounds such as a very steep hill for which in order to place the four legs on the ground the hill top would break through the table top.

This was achieved through four successive uses of the I.V.T., which was complicated by necessity of the uniqueness of each stage to show continuity of motion in the final rotation. So for square or rectangular tables, given a ground that is not too wild, a table should balance by turning roughly about the centre. However the table may not be horizontal, since we don't want our beer to slide off the picnic table is it possible to balance the table horizontally? In

Fenn's paper this is proven possible given certain restrictions on the ground. Paraphrased the theorem says:

Theorem: If a continuous ground coincides with the xy-plane outside a compact convex disc and if the ground never dips below the xy-plane inside the disk, then a given square table can be balanced horizontally such that the centre of the table lies above the disk. The given restrictions can be thought of a hill with a boundary.

In each paper it was discussed what other shaped tables might be possible to balance. Each concluded a three-legged mathematical table was possible to balance both locally and horizontally, the horizontal case was proven by Zaks for the ground functions described by Fenn. For tables of five legs and above it is not generally possible to balance the table locally or horizontally since five points uniquely defines a conic section and it would be possible to create a continuous function that did not contain a copy of the conic described by the table in question.

It has also been shown that it is necessary for a four legged table that the legs are concircular, however it is open as to whether concircular tables other than the square and rectangle can be locally balanced or tables other than the square can be horizontally balanced. I intend to look further into this problem as part of my honours project.

The project was an invaluable introduction to my honours year and I'm very grateful to my supervisor Burkard Polster for his patience and enthusiasm. I am also grateful to AMSI for making the opportunity available.

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See www.ice-em.org.au/students.html#scholarship.