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Formation of Black Holes in Spherically Symmetric Spacetimes with Scalar Field

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The aim of this project was to read and explain a paper by Demetrios Christodoulou on the formation of singularities in a spherically symmetric spacetime in the presence of a scalar field. We used Cartan's method of moving frames to perform the calculations.

A singularity is basically a point or region of spacetime where the measurable quantities on relativity such as curvature become infinite. Some such singularities result from inappropriate choice of coordinate systems and thus can be removed by a different choice of coordinates. However some are invariant under change of coordinates and thus cannot be removed. The Schwarzschild solution describes such a singularity for a spherically symmetric spacetime in a vacuum. As this is the only solution to the Einstein equations in that simple situation, Christodoulou considers the slightly more complicated problem of a spacetime with the presence of a real massless scalar field. He shows that in terms of Bondi coordinates the metric in this situation becomes:

$$g_{ab}dx^a dx^b = -e^{2\nu}du^2 - 2e^{\nu+\lambda}dudr$$

In the above metric u is the null coordinate and r the radial coordinate.

Cartan's method is a very effective way to calculate curvature and involves using the following relations between the Riemann tensor and the curvature two forms:

$$\Omega^a{}_b = \frac{1}{2}R^a{}_{bcd}\,\omega^c \wedge \omega^d$$

$$\Omega^a{}_b = d\Gamma^a{}_b + \Gamma^a{}_c \wedge \Gamma^c{}_b$$

The above is given in an orthonormal basis and the change of variables into a coordinate basis is given by

$$R^i{}_{jkl} = (\Lambda^{-1})^i{}_a R^a{}_{bcd}\Lambda^b{}_j \Lambda^c{}_k \Lambda^d{}_l$$

Where $\Lambda^i{}_a$ is the transformation matrix, a b c and d are the orthonormal indices and I j k and l are the coordinate indices.

Although I did not get as far into it as I would have liked this project taught me a great deal about relativity and differential geometry.

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See http://www.ice-em.org.au/archive.html#vacation2005.