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Functional analysis: investigations in Costa's surface
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My vacation involved investigating the paper 'A complete embedded minimal surface in $\Re^3$ with genus one and three ends', by DA Hoffman and W Meeks III. The surface in question is more commonly known as Costa's Surface, in honour of its discoverer. It is a minimal surface constructed using a technique known as Weierstrass' representation, which produces a correspondence between minimal surfaces and holomorphic and meromorphic complex functions.

The first part of the project was to investigate the construction and proof of theorems concerning Weierstrass' representation using the referenced text by Pressley (2001). Next, three examples of minimal surfaces constructed using Weierstrass' representation were given in Hoffman and Meeks, and it was verified that the surfaces given were the surfaces claimed, noting that sometimes the Weierstrass representation formula can give a parameterisation for a surface that differs greatly from the 'standard' parameterisations for these surfaces given in lecture courses (the catenoid is a good example of this).

Weierstrass' representation formula involves starting with a holomorphic complex-valued function *f(u,v)* and a meromorphic complex valued function *g(u,v)* on an open subset U of the complex plane, where *f* and *g* satisfy specific given conditions. From these functions, a vector valued holomorphic function φ**(u,v)** in 3-space is created and then the real part of a complex line integral over φ is the required minimal surface σ**(u,v)**. To produce the parameterisation for the Costa surface, the functions *f(u,v)* and *g(u,v)* which are used in the Weierstrass representation formula depend on a function known as the Weierstrass $\wp$-function. As such the second part of the project involved investigating the properties of this function and functions of its type to show that the holomorphic and meromorphic functions produced using the $\wp$-function satisfied the conditions for Weierstrass' representation, therefore showing that the Costa surface is indeed a minimal surface.

In the six weeks over which the project took place, I was able to study about a third of the paper by Hoffman and Meeks, with the rest of the paper concerned with how the symmetries of the $\wp$-function can be effectively used to show symmetry properties of the Costa surface, and to prove the claim in the title of the paper, that the Costa surface is of genus one with three ends.

References:

1) DA Hoffman, W Meeks III. **A complete embedded minimal surface in $\Re^3$ with genus one and three ends.** J. Diff Geom **31** (1985) Pg 109-127
2) A Pressley. **Elementary Differential Geometry.** Springer Verlag London (2001)
3) ET Whittaker, GN Watson. **A Course of Modern Analysis (4th Ed.).** Cambridge University Press (1958)

Stephen received an ICE-EM Vacation Scholarship in January 2005.
See www.ice-em.org.au/students.html#scholarship.