The commuting graphs of groups

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The commuting graph of a group is the simple undirected graph whose vertices are the non-central elements of the group and two distinct vertices are adjacent if and only if they commute. The properties of these graphs, such as their connectedness and diameter, have recently been examined in literature. Iranmanesh and Jafarzadeh conjectured that there is a universal upper bound on the diameter of the commuting graph of any finite nonabelian group if that graph is connected [1]. Many examples in the literature support this conjecture. My aim was to investigate some more examples of finite groups and see if the conjecture held or could be disproved.

I decided to investigate the group of 2*x*2 invertible matrices over the ring of integers modulo *m* for some integer *m*. The case of *m* being prime and then the ring being a field was already covered in prior literature. By examining specific examples through use of Mathematica and Magma, I was soon able to prove that if *m* were a prime power then the diameter of the commuting graph of the group was 3 and, after some more arduous effort, that if *m* were any composite then the diameter was exactly 3. I then extended this result to show that, for any *n* and composite *m,* the group of *nxn* matrices over the integers mod *m* has a connected commuting graph with a diameter of exactly 3.

The presentation of the topic and my results were well received at the CSIRO Big Day In. It was also great to network with my peers. I enjoyed immensely working on my own problems and coming up with my own proof for things. This project has given me insight into what research in maths is really like and motivated me to pursue Honours and possibly a PhD in Pure Mathematics. I would like to thank CSIRO and AMSI for this unique opportunity. I would also like to express my deep thanks to my supervisor, Dr Michael Giudici. Michael entertained my more than daily visits to his office and continually motivated me to keep trying to approach the problem from different angles, sacrificing much of his time in the process. We aim to publish the results in a small paper soon, and I hope to continue to work on the conjecture and related questions about the commuting graphs of groups in the future.

References

[1] A. Iranmanesh and A. Jafarzadeh, 'On the commuting graph associated with the symmetric and alternating groups'. Journal of Algebra and its Applications. 7 (2008), 129-146.

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