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Slowly Varying Culling in a Single-Species Population Model

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In many population models describing single species, the population tends to a limiting value representing the total population the environment is capable of supporting. This value, a constant, is termed the *carrying capacity* of that species in that environment. Such behaviour is typical of the solutions of the logistic and Gompertz models, to name just two.

In such models, a further complication arises when the species is *culled*, that is, population is removed at a given constant rate. In such a case increase of the cull rate above a certain value leads to the extinction of the species; i.e., the population reduces to zero in finite time. In a real situation, however, both the carrying capacity and cull rate may not be constant, but may vary over time. Background features in the environment, such as rainfall, tidal variation, seasonal variation of food sources all cause the carrying capacity to vary; while variations may occur in the culling agent itself. Typically, this variation may be expected to be relatively slow, so that the overall population variation may be viewed as depending on two time scales – normal time, and a slower time scale, characterizing the variation in carrying capacity and culling. Thus, it might be expected that such population modelling problems could be analysed using a multitiming approach, based on these two scales.

As an initial calculation, we solved the governing differential equation when the carrying capacity, birth rate and culling rate were constants, and used Maple to obtain plots of the varying population. We found that a too high culling rate or a too low initial population could lead to the population becoming extinct. However, if the population were started at a sufficiently high number, the resulting population would tend to a constant value.

We next extended this calculation to a varying culling rate, with fixed birth rate and carrying capacity. In this case, the exact solution for the governing differential equation could not be obtained, due to the varying culling. However, for culling varying slowly, the multitiming approach could be applied, to obtain an explicit approximate expression for the population. Multitiming proved a good method to use, because the approximate multitiming solution agreed very well with the numerical solution of the problem. We could use this approximate solution to plot the population's behaviour. Again, the population became extinct when the initial population was too low or the (varying) culling was too high, while the population survived for a high enough initial population.

At the moment, we are continuing working on the case when the culling and carrying capacity are varying, but the birth rate is fixed. Furthermore, we will also look at when culling, carrying capacity and birth rate are varying, in more detail.

Being an AMSI scholar provided me with an opportunity to communicate with others outside my school, by presenting my project through the oral presentation and a report. This helped me to build my confidence, communication skill, writing skill and time management.

Linda received an ICE-EM Vacation Scholarship in December 2005.
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