Quantum Structures of Yang-Mills Fields

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My summer research project was called *Quantum Structures of Yang-Mills Fields*. Yang-Mills theory is extremely important in modern theoretical physics as it provides a mathematical description of the behaviour of the force-carrying "gauge bosons" in the Standard Model of particle physics. The theory is well understood in the framework of classical physics, and has lead to many interesting results such as the proof of existence of exotic differentiable structures on R^4, however there is currently no corresponding quantum theory.

The difficulties in obtaining a quantum theory arise, in part, because most of the known quantization techniques neglect the rich geometric structure which forms the basis of the classical theory. Because of the prominence of Yang-Mills theory, the problem of quantization is quite important, in fact showing the existence of a quantum Yang-Mills theory in four-dimensional spacetime forms part of one of the Clay Mathematics Institute's Millennium Prize Problems.

The aim of my project was to investigate possibilities for a framework for a quantum Yang-Mills theory which preserves the elegant geometry present at the classical level. This was done primarily through a review of current literature on the jet-theoretic formulation of classical field theories – where the dynamics of the theory are described on the space of partial derivatives of the field variables: the *jet bundle* of the configuration space – as well as some of my own work centering on constructing a "dual jet-theoretic" formulation.

This was done because the canonical approach to quantization uses a special formulation of classical particle mechanics known as the *Hamiltonian* formalism. For relativisitic field theories, such as Yang-Mills theory, the standard Hamiltonian formalism is insufficient to describe the whole dynamics of the theory as it spoils some of the relativistic symmetries of the theory.

My main goal was to develop a generalization of the Hamiltonian formalism appropriate for describing relativistic field theories, such as Yang-Mills theory, which preserves relativistic symmetries as well as any underlying geometry that might be present in the classical theory. This was done by "dualising" the standard jet-theoretic description of field theories using the notion of duality for affine spaces. This turns out to be one method of generating a *multisymplectic geometry*, which is in some sense the field-theoretic generalization of *symplectic geometry* which is used in the Hamiltonian formulation of classical particle mechanics.

There are some open problems with using multisymplectic geometry to describe a field theory. Since we ultimately want to obtain a quantum theory, we want our classical theory to be formulated in a way that makes the transition to quantum theory easier; this role is played by the Hamiltonian formulation and symplectic geometry in the case of classical particle mechanics. The problem with multisymplectic geometry, however, is that it is not a perfect generalization of symplectic geometry because some of the crucial properties of the theory aren't readily apparent.

Recent research in this particular area has focused on refining multisymplectic geometry in order to address the above problem, and there are a few alternative solutions each with its own advantages and disadvantages. In the final stages of my project, I started developing another method of treating multisymplectic geometry which is intimately related with the structure of spacetime and which was motivated by an aesthetic appreciation of the elegant geometries involved at the classical level. This method promises to be an interesting area for more research, and in some sense resembles the underlying mathematics of Loop Quantum Gravity.

In summary, this project has been an extremely valuable learning experience; it gave me an excellent opportunity to learn about recent advances in mathematics and about the research process. It has enhanced my interest in mathematics and academic research and it has allowed me to pursue some of my own mathematical interests. Overall, it was a great opportunity which should prove to be extremely beneficial for my honours year.

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