Sensitivity to Initial Conditions: Characterizing the Accuracy of the Weather Forecast

Kate Snow
University of Wollongong

The aim of this project is to investigate how errors in chaotic numerical weather models affect the predictions of the weather. Due to the highly complex nature of weather models, the Lorenz equations with their chaotic nature are chosen as a toy model in which errors in initial conditions will be investigated. Direct atmosphere applicability of the Lorenz equations is not important since actual atmosphere, with its turbulent behaviour, must still be sensitive to initial conditions.

To investigate this issue, instead of looking at time series plots of a single variable in the chaotic region and making graphical comparisons as initial conditions are perturbed (requiring 1000's of visual comparisons), a time scale is divided into "days". Statistical behaviour (maximum, minimum, mean and standard deviation) of the time series of each day are calculated and compared. Plotting the results of these statistics for a certain error range of initial conditions allows determination of how each day's results varied compared to the next, giving an idea of predictability of the days. Predictability is defined by limiting standard deviations of the factors calculated over the initial condition range for each day, values with standard deviations below that limit illustrate less variability of the results and hence some greater form of predictability to those with higher standard deviations.

Predictability of day 1 is found with a 0.1% error around the initial condition 2, higher errors gave unpredictable results. Day 2 does not show predictability until an error of 10^{-9}% indicating an exponential trend. These results and the exponential trend can be verified using the equation;
$||\delta(t)|| \sim ||\delta(0)||e^{(0.9t)}$
This exponential increase in errors as time progresses illustrates how sensitive dependence makes long-term forecasts of the weather impossible.

This project was of great benefit allowing development of key skills such as understanding of mathematical modeling, ability to analyze large groups of data using statistics, etc that would be beneficial in future endeavors. Of particular benefit was the chance to attend the Big Day In, which provided access of new and exciting areas being researched. It also allowed development of presentation skills, as well the chance to socialize with peers and industry representatives and professors within areas that could potentially provide future career paths. I would like to thank AMSI for this experience.

Kate received an AMSI Vacation Scholarship in December 2010 See: www.amsi.org.au/vs10.php

Postal Address: 111 Barry Street
c/- The University of Melbourne
Victoria 3010 Australia

Email: enquiries@amsi.org.au
Phone: +61 3 8344 1777
Fax: +61 3 9349 4106
Web: www.amsi.org.au