INTERNATIONAL CENTRE
OF EXCELLENCE FOR
EDUCATION IN
MATHEMATICS

Real Shabat Polynomials.

Edward Ross, School of Mathematical Sciences, The University of Adelaide

A real Shabat polynomial is a real polynomial P of one variable with no more than two critical values, such that for each critical value c, all the roots of the polynomial P - c are real.

The main focus of my vacation scholarship was to classify and investigate some properties of the real Shabat polynomials. I also undertook a side project to understand the fundamental concepts of the size of infinite sets.

I found that if P(x) is a real Shabat polynomial, then so is kP(mx-c)+d where k,m,c and d are real numbers, and k and m are not zero. This leads to the idea of partitioning the real Shabat polynomials into equivalence sets, where two polynomials P(x) and R(x) are equivalent if there exists real numbers k,m,c and d, with k and m not zero, such that R(x)=kP(mx-c)+d.

I also found that real Shabat polynomials with one or zero critical values are equivalent to the monomial xn where n is a positive integer. I discovered that representatives for real Shabat polynomials with two distinct critical values can be found by solving the differential equation
n2(P(x)2-1)=P'(x)2(x2-1),
for integers n≥3.

The differential equation has a unique polynomial solution for each n≥3, the nth Chebshev polynomial Tn(x)=cos(n arccos x).

The summer research scholarship allowed me to undertake actual research in mathematics, which was both a frustrating and rewarding experience. I learnt much about researching and about writing formal mathematical reports. Attending CSIRO's Big Day In gave me experience in communicating mathematics, and gave me the opportunity to meet many interesting people studying a broad range of mathematics, which was fantastic.

Edward received an ICE-EM Vacation Scholarship in December 2006.
See http://www.ice-em.org.au/students.html#scholarships2007