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Introduction to C*-algebras

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The axioms of C*-algebras abstract the properties of the space of all bounded linear operators on a Hilbert space, and lead to many applications, including in physics and differential equations. I worked with Paul Williams and Matthew Young for this vacation project with A/Prof Jerry Koliha as our supervisor as we were introduced to C*-algebras and over the course of our project studied various topics in Banach and C*-algebras.

An algebra is a complex vector space with an additional multiplicative operation. A Banach algebra is an algebra that is normed with a submultiplicative norm and complete with respect to this norm, such that it is also a Banach space. A C*-algebra is a Banach algebra with the addition of an involution operation that satisfies the C*-Identity: $||a||^2 = ||a^*a||$.

We investigated different aspects of these abstract algebras and proved some interesting results, both elegant in their own right while also being important properties of the algebras. A significant part of the development of C*-algebras is the proof that $(1+a^*a)$ is invertible for any a. Another important result is that a C*-algebra can be equivalently thought of as a space of bounded linear operators on a Hilbert space, which allows the application of results from operator theory to prove things about arbitrary C*-algebras.

I found the vacation scholarship to be well worth undertaking and would recommend it to anyone with an interest in studies beyond undergraduate level in mathematics. In addition to studying some interesting mathematics, we also developed skills that should prove invaluable in future studies such as familiarity using LaTeX and presenting complete proofs. I am very grateful to my team members, our supervisor and all the vacation scholars at Melbourne University that I had the chance to interact with for making this a rewarding experience.

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See http://www.ice-em.org.au/students.html#scholarships2007