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Estimating the tine change of Brownian Motion

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In the world of financial mathematics there is one equation which reigns supreme: the nobel prize winning '*Black-Scholes Option Pricing Formula*', developed in 1973. Even as widely used as the equation still is today, the past 34 years has provided a plethora of empirical evidence which casts serious doubts on its assumptions. Central to these doubts is the underlying assumption that asset prices evolve like a 'Geometric Brownian Motion' (GBM).

The race is currently on to find a mathematical model for asset prices which will supersede GBM and give us a more realistic framework to analyse financial markets.

My project supervisor Dr. Owen Jones is currently researching the validity of one such model, known as 'Time Changed Brownian Motion' (TCBM). The key idea is that instead of forcing time to tick by at a constant rate, we allow it to speed up and slow down as the market goes through periods of frenzied hype and quiet inactivity. This 'market time' gives the model the flexibility to account for the moody nature of the market while still retaining many of GBM's more useful features.

Dr. Jones had already determined that TCBM was a statistically viable model for several year-long foreign exchange financial time series, and gave me the job of estimating what exactly the time change was for this data, and also testing some of the statistical properties of this estimated time change.

After reading some of the recent literature on the subject I slowly built a Matlab program step by step until eventually it was able to reliably handle the large volume of data it was required to process (in a year the EUR/USA exchange rate changes about 7.5 million times!), and produce all the necessary output.

The results I found suggested that market time varied considerably from clock time, and also showed that the time change is itself a 'Long Range Dependent' process – a very desirable result!

For me the best part of the project was working on a cutting edge area of mathematics with no idea what results I would find. I would strongly encourage anyone with a suitable background in mathematics and a curious mind to apply for the AMSI summer program.

Jonathan received an ICE-EM Vacation Scholarship in December 2006.
See http://www.ice-em.org.au/students.html#scholarships2007