Tropical Geometry
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Over the 2006/07 summer, I had the privilege of working on a research project as part of the vacation scholarship program at the University of Melbourne, under the supervision of Associate Professor John Groves. The topic I researched was tropical geometry – algebraic geometry with a tropical twist.

Tropical algebraic geometry is the geometry of the tropical semiring. This is the set of real numbers (typically augmented by infinity) with the basic operations of arithmetic redefined. We define tropical addition and tropical multiplication as follows:

$$a \oplus b := \min\{a, b\}$$
$$a \odot b := a + b$$

From here, we can form tropical Laurent polynomials with coefficients in our tropical semiring and then look at the geometric objects that arise when investigating these piece-wise linear concave functions. The tropical hypersurface of a tropical polynomial is one such object. It is defined as the set of all points where the minimum is attained by at least two monomials. As an example, the tropical hypersurface of

$$F = 0 \odot x \oplus 0 \odot y \oplus 0 = \min\{x, y, 0\}$$

is a tropical line consisting of three half-rays emanating from the origin in northern, eastern and southwestern directions.
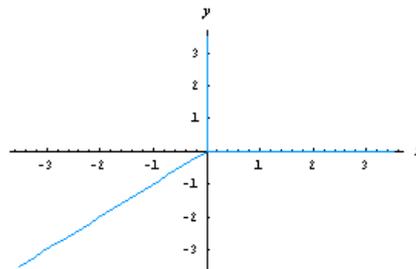
Tropical hypersurfaces are examples of more general objects called tropical varieties. In fact, it turns out that every tropical variety is the intersection of finitely many tropical hypersurfaces. The converse though is not true.

In addition to studying the general theory and performing a literature search, I also worked on an interesting problem that investigated symmetry groups of tropical varieties. The problem was to determine which finite subgroups of invertible integer matrices occur as automorphism groups of tropical varieties. For instance, the tropical line has D_3 symmetry (there are 6 possible ways to permute its three half-rays via matrix multiplication).

Overall, I found the vacation scholarship program to be a truly rewarding experience. It provided great insight into the research side of mathematics, especially into a relatively new field. I am also grateful to AMSI and CSIRO for giving me the opportunity to attend the Big Day In and present a short talk on my project.