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Properties of a special Fuchsian Differential Equation

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I spent December 2006 and early January 2007 making friends with a certain ordinary differential equation that my supervisor, Professor Peter Forrester had come across in his research in random matrix theory.

This all happened because Prof. Forrester had met some useful functions in his mathematical travels that were expressed in the form of N-fold integrals. Unfortunately even evaluating functions in this form, for even moderate values of N, is a very computationally complex process.

Fortunately these functions happened to satisfy a differential equation and methods for finding series solutions, which are far more practical to work with, for such equations are well known.

So the main aim of the project was to find (in general) these series solutions to the differential equation and relate them to the original N-fold integrals.

This involved learning quite a bit about general linear ordinary differential equations, and some MAPLE programming.

Whilst doing this I often become restless and wondered about the deeper structure and properties of the differential equation. For example, what is so special about this differential equation that means it has integral solutions? This lead me to do some reading in the area of Differential Galois Theory which extends ideas from Galois Theory to linear homogeneous ordinary differential equations.

The project gave me some confidence in, and aroused plenty of interest in an area of mathematics of which I had little understanding beforehand. Together with the advice and leadership of my supervisor (who was a super supervisor), the project gave me some sense of what it is like to do research in mathematics. As such it was a very valuable opportunity that I am most grateful to ICE-EM/AMSI and the University of Melbourne for providing.

James received an ICE-EM Vacation Scholarship in December 2006.
See http://www.ice-em.org.au/students.html#scholarships2007