Lévy Processes

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Lévy processes are a class of continuous-time stochastic processes possessing stationary and independent increments. As they form a tractable family of stochastic models with jumps, they have found applications in a number of areas such as finance, physics and biology.

The first part of my project involved studying the theory of Lévy processes where I derived alternative proofs for some of the key results. I referred to the book by Peszat and Zabczyk (2007) which provided a functional analytic perspective on the theory. The main topics covered were:

1. *Lévy–Khinchin decomposition and formula* – first interpretation of a Lévy process as a superposition of a Wiener process and a number of compound Poisson processes.
2. *Transition semigroups and infinitesimal generators* – characterisation of a Lévy process as a Markov process that is homogenous in space and time
3. *Poisson random measures* – second interpretation of a Lévy process as an infinity activity process that moves by an infinite number of small jumps
4. *Stochastic integration* – construction of stochastic integrals with respect to a Lévy process like a Cauchy process.

For the second part of my project I focussed on some applications outlined in the papers by Garbaczewski and Olkiewicz (2000) and Masuda (2004). Both notes analyse the Ornstein-Uhlenbeck (OU) process $\{V_t\}_{t\geq 0}$ driven by a Lévy process $\{L_t\}_{t\geq 0}$ in the one-dimensional and multidimensional setting respectively. The Lévy OU process satisfies the following stochastic differential equation which represents the velocity process of a particle in a fluid.

$$\mathrm{d}V_t = -\Lambda V_t \mathrm{d}t + \mathrm{d}L_t$$

The displacement process $\{X_t\}_{t\geq 0}$ is then given by the integrated Lévy OU process defined below.

$$X_t = X_0 + \int_0^t V_u \mathrm{d}u$$

Based on these papers, I established that the displacement process was not a Markov process in contrast with the claims made by Garbaczewski and Olkiewicz.

Overall, this project was an illuminating experience which I would highly recommend to new students. As such, I would like to thank UNSW, AMSI and MASCOS for their financial support.

References

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Carl was an AMSI/MASCOS Vacation Scholar for 2007-8.