INTERNATIONAL CENTRE OF EXCELLENCE FOR EDUCATION IN MATHEMATICS

Fractional Brownian Motion

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The stochastic process known as Brownian motion has been used in a wide range of fields over a period of time, stretching from physics and quantum mechanics to mathematical finance and security markets modelling. The particularly attractive properties of this process are derived from considering the interaction of the process with itself over a period of time, since a standard Brownian motion is Markovian (i.e. the value of the process at any time *t* depends only on the situation at time *t*, and is independent of past behaviour, for example flipping a fair coin).

In the real world, however, there is often noticeable time-dependence in systems, and so this assumption is flawed. One simple alternative, which encapsulates this time-dependence AND offers a number of the other desirable qualities of the standard Brownian motion (e.g. its Gaussian behaviour) is the so-called fractional Brownian motion, which builds on the standard Brownian motion, and is associated with quite a different covariance structure.

During the course of this six-week project, I looked at some of the theory surrounding fractional Brownian motion processes, and in particular parts of the theory encapsulated in the papers 'Stochastic Analysis of the Fractional Brownian Motion'(written by L. Decreusefond and A.S. Üstünel[1]), 'On Fractional Brownian Processes' (written by Denis Feyel and Arnaud De La Pradelle[2]), and 'Ergodicity of Stochastic Differential Equations Driven by Fractional Brownian Motion' (preprint produced by Martin Hairer in September 2004).

This project was very rewarding, and provided valuable insights into the way in which mathematical research may be conducted in practice, as well as offering an opportunity to explore areas of mathematics that can only be touched on briefly - if at all - in the limited time available during a teaching year.

[1]Published in 1999 in ***Potential Analysis 10***: *177-214*.

[2]Published in 1999 in ***Potential Analysis 10***: *273-299*.

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See www.ice-em.org.au/students.html#scholarship.