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Identifying Macroscopic Structure in Dynamical Systems

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For a dynamical system, described by a mapping S, defined on a phase space $X, S : X \to X$, a set $A \subset X$ is called an **invariant set** if $S^{-1}(A) = A$. This implies that S maps A into itself and no element of $X \setminus A$ is mapped into A. When given a dynamical system we would like to be able to decompose the phase space X into a number of invariant sets, but they may not always be easily found, or in some cases they may not even be possible.

In my project we studied almost-invariant sets, where *almost* means that it is very probable (rather than definite) that trajectories in an almost invariant set will remain in it for a sufficient period of time. Since the trajectories are mostly contained inside these sets over a period of time, they represent what we would identify as the macroscopic structures of the dynamical system.

During the course of the six-week project, I used a software package GAIO (Global Analysis of Invariant Objects) with MATLAB to find the stable and unstable manifolds of various dynamical systems, including a slight nonlinear modification to the famous Anosov Diffeomorphism known as the 'Cat Map' and the double pendulum system. Using the software I found eigenvalues to a discrete approximation of the Perron-Frobenius Operator and we made some attempts to try and work out why the eigenvectors graphically bear such a strong resemblance to the unstable manifold. In attempting to answer this question, I also looked at Markov partitions and Symbolic dynamics, but the direct correspondence remains a mystery.

Having travelled from interstate, it was fascinating to see how applied mathematics is done at UNSW and I thank my supervisor Dr. Gary Froyland for all his valuable help.

For other students interested in dynamical systems, a good, readable introduction to some of the work I looked at is found in:

Lasota, A. & Mackey M C., *Chaos, Fractals, and Noise - Stochastic Aspects of Dynamics*, Springer-Verlag, 1994.

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See www.ice-em.org.au/students.html#scholarship.