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**Generalising the Fourier Transform**

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Banach algebras are abstract structures that occur in the study of analysis and algebra, and blend the 2 disciplines naturally. The Gelfand-Naimark Theorem provides a connection between (a class of) these abstract Banach algebras and a space of functions, which we can work with relatively easily. In working up to this theorem, one needs to deal with the Gelfand Transform, which coincides with the Fourier Transform when on the familiar spaces.

Studying the Fourier Transform and its intermediate generalisations is therefore beneficial in developing a deeper understanding of this Gelfand Transform (on commutative unital C*-algebras). The generality of the Gelfand Transform makes it easy to define, as opposed to the Fourier Transform for locally compact abelian (LCA) groups. As a result, the project focuses mainly on the development and construction of a sufficient definition of the latter.

The Fourier Transform can be defined on the characters of a group, and this is the minimal structure that gives rise to the (desired and required) multiplicative property of the transform, but such a definition lacks the practicality of the classical Fourier Transform on $\mathbf{R}^n$. In developing a much more practical (yet equivalent) definition, we require the concept of a dual group. It turns out that the characters of a group can be identified with the (usually simpler) dual group, on which we can define the Fourier Transform (practically). Some familiar examples, such as the Fourier Transform on the integers and on the circle group, were considered.

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See www.ice-em.org.au/students.html#scholarship.