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The geometry of hyperbolic platonic surfaces

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I did my vacation project under the supervision of Michael Cowling. It was different to how I had ever approached studying mathematics before by having no obvious path of knowledge to journey down. This was some incite into research mathematics and was very rewarding but also gave me appreciation of how much easier learning is when structured in lecture format or even self-study from a book.

I was trying to understand a family of abstract surfaces sometimes referred to as “Modular Surfaces”. We get this name as they are the constructed be quotienting out the hyperbolic plane by the action by the modular group $\Gamma(N)$. This is the group of integer matrices a, b, c, d such that $ad-bc=1$, $a=d=1 \pmod N$, and $b=c=0 \pmod N$.

The first way to attack this problem was to figure out the topology of each surface. I had success here using general theorems (eg the generalised Gauss-Bonnet Theorem) from differential geometry combined with knowledge of the modular group. I found formulae for the number of cusps, the genus.

The next stage was trying to find the fundamental domains for each surface. I gave up finding a nice way after reading how the two approaches other people had used needed trial and error and many cups of coffee. I had to be content understanding this for small N.

Throughout the process I stumbled into many dead-ends but also had the excitement of something working without previously being told by someone else that it would work. This unknown end is no problem when passion is there. Furthermore it allows for “crazy” ideas to be considered. I thoroughly enjoyed this summer and am very grateful for the scholarship and to Michael.

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See http://www.ice-em.org.au/students.html#scholarships2007