Modeling dependence in operational risks using copulas and common factor models

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During the early stages of the project, I learnt about using WinBUGS for parameter estimation using Markov Chain Monte Carlo methods. This gave me an opportunity to learn about how these methods work as well as the variety of problems they can be applied to.

Following this, we began to consider operational risk models and different ways to model dependence between operational risks. This is important since the presence of dependence between operational risks will affect the way these individual risks are aggregated when considering the total operational risk of the business. The model used for an annual loss for year t in a risk cell, j, is a compound random variable:

$$Z_t^{(j)} = \sum_{s=1}^{N_t^{(j)}} X_s^{(j)}(t)$$

Where, for example, $N_t^{(j)}$ is the number of losses for the risk cell and has a Poisson distribution with parameter $\Lambda_t^{(j)}$ while $X_s^{(j)}$ is the severity of each loss and has a log-normal distribution with mean zero and standard deviation parameter $\Psi_t^{(j)}$.

Three different ways of modeling dependence between operational risks were considered:

1. Inducing correlation between the frequency and severity parameters ($\Lambda_t^{(j)}$ and $\Psi_t^{(j)}$ respectively) using common factor models
2. Using copulas to induce correlation in the frequency and severity parameters. The t-copula was used along with the Gumbel, Clayton and Frank copulas which are part of the Archimedian family of copulas
3. Inducing correlation between the frequency parameters using a common factor model while using a copula for the correlation between the severity parameters

We then used simulations to gain some insight into the level of correlation in the annual losses that could be induced using each of these models. Generally we found that

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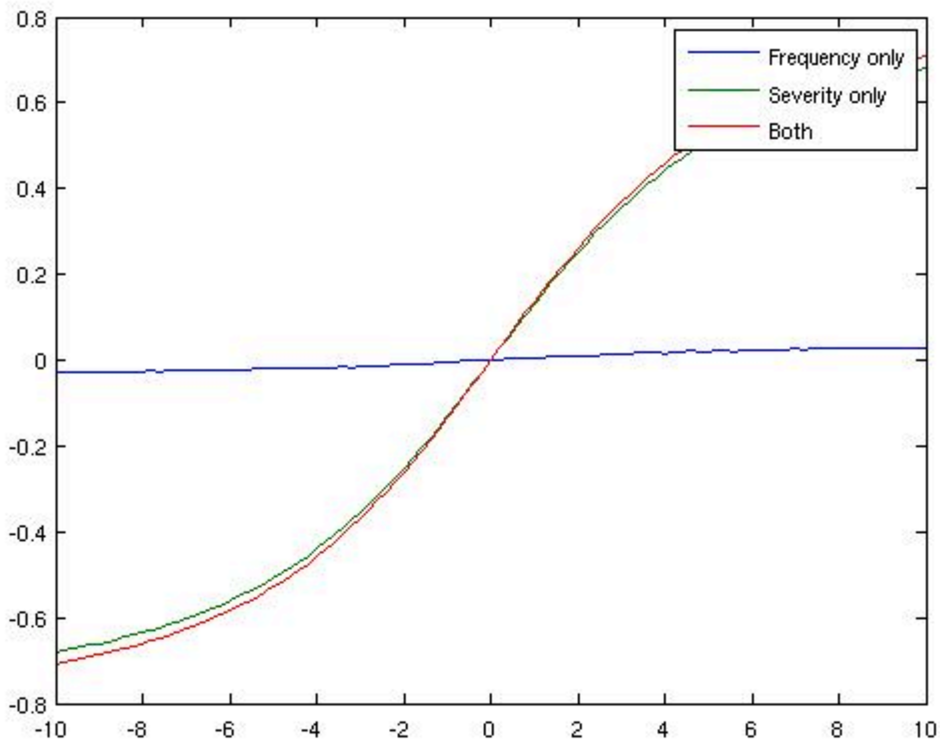
inducing correlation between the frequency parameters resulted in low levels of correlation in the annual losses while inducing correlation between the severity parameters resulted in high levels of correlation in the annual losses. As an example, a graph is included on the following page showing the levels of correlation that were induced using the Frank copula in method 2. Correlation is on the vertical axis and is measured using the Spearman's rank correlation coefficient while the copula parameter is on the horizontal axis.

From this point, it is possible to construct a Bayesian model for the parameters and this could be used to estimate the parameters of the model given actual loss data via Markov Chain Monte Carlo methods. Another possible direction for future work is to look at alternative model specifications in which correlation in the frequency parameters may result in higher levels of correlation in the annual losses.

In closing, I found undertaking this project to be a valuable and worthwhile experience. I would like to take this opportunity to thank AMSI and UNSW for their support along with my supervisor Scott Sisson and Gareth Peters.



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