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Semi-Classical Analysis of a Bose-Einstein Condensate Model

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Bose-Einstein condensates are macroscopic systems consisting of particles all in the same quantum state. Although they must be treated using quantum mechanics, one can derive a classical Hamiltonian to approximately describe the system and hence use the Hamiltonian formulation of classical mechanics to investigate the time evolution of the condensate.

Condensates can be created with mixtures of atoms and molecules in which a chemical reaction is occurring. For my project, I investigated the classical dynamics of an atomic-molecular Bose-Einstein condensate model that takes into account internal scattering interactions.

Initially, I reproduced the results for the case of a condensate with a single atomic mode and molecular mode. This technique was then extended to consider a three-mode condensate containing two different atomic modes and a molecular mode.

Starting with a quantum mechanical Hamiltonian describing the three-mode system, a classical Hamiltonian was derived. The classical Hamiltonian was found to have a three-dimensional parameter space but only two time-dependent quantities – the relative phase between modes of the system and the fractional proportion of molecules present.

Hamilton's equations were used to derive the equations of motion or time evolution of the system. Where there was an equal occupation of the two different atomic modes, the qualitative dynamics were identical to a condensate with one atomic and one molecular mode. However, the dynamics differed significantly for this system where there was an excess of one type of atom present.

In the latter case, the three-dimensional parameter space could be divided into two distinct regions of dynamical behaviour based on the number of fixed points of the system. A combination of asymptotic approximations and numerical results were used.

Level curves of the Hamiltonian, which correspond to trajectories of the system, were plotted for different sets of parameters in order to visualize the dynamics. Parameters were chosen from both regions identified in the parameter space. In general, the proportion of molecules in the system oscillated over time whilst examples of both running and oscillating phase were observed.

I found the vacation scholarship a rewarding and enjoyable experience that I would recommend to anyone who enjoys mathematics. It has given me a valuable insight into research in mathematics in the lead up to my honours year.

REFERENCES

[1] G. Santos, A. Tonel, A. Foerster and J. Links. Classical and quantum dynamics of a model for atomic-molecular Bose-Einstein condensates. Physical Review A 73 (2006) 023609

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