Cycle Decompositions of Graphs
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Under the supervision of Darryn Bryant, I studied and worked on some cycle decomposition problems which Darryn and his research group have been researching recently. A cycle decomposition is a collection of cycles whose edges partition the edges of a graph. An m-cycle decomposition contains only cycles of length m.

In my research I investigated cycle decompositions of circulant graphs with edges of distance 1, 2 and 3. That is, the graph obtained from the n-cycle by joining vertices at distance 2 and 3 (equivalently, the Cayley graph $Cay(Z_n, \{1,2,3\})$). In particular, I completed the final cases needed for a proof of a result that these graphs have m-cycle decompositions for all $3 \leq m \leq n$ when m divides 3n. Whether or not these graphs have decompositions into cycles of lengths $m_1, m_2, \ldots, m_t$ whenever $m_1 + m_2 + \ldots + m_t = 3n$ is an open problem.

I also worked on the circulant graph with edges of distance 1 and 2. It is known that there are many cycle decompositions that these graphs do not admit. The problem I solved was to describe exactly which decompositions they do admit.

I found that my topic was a great choice for a vacation scholarship because I got the opportunity to actively participate in current research. I enjoyed this more than I would have enjoyed just reading about the area.