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Class Groups of Quadratic Number Fields
Thomas McCourt, Department of Mathematics, University of Queensland

I spent my AMSI scholarship investigating p-adic numbers. The majority of the work I completed was based on the text '*p-Adic Numbers: An Introduction*' by Gouvea (second edition; Berlin, New York: Springer c1997).

I began by investigating the solving of congruences modulo p^n for some prime p. If we define $\mathbf{Q}_p$ to be the set of finite tailed Laurent sequences in powers of p, or rather, the field of p-adic numbers, then solving these congruences corresponds to solving of the corresponding equation in $\mathbf{Q}_p$.

In order to move on in this area I had to first learn some background theory in the abstract theory of absolute values on a field, and more generally to the study of topological spaces.

Once I had this background I was able to move onto absolute values on **Q**. This included some very interesting results, such as Ostrowski's theorem that every non-trivial absolute value on **Q** is equivalent to either a p-adic valuation or the usual (archimdean) absolute value (the so called prime at infinity). These valuations are all related by the product formula: for any $x \in \mathbf{Q}^*$, we have $\Pi Ł x Ł_p = 1$, where we take the product over all the primes of **Q** including infinity.

I now needed to find out about completions of a field. A field is complete if every Cauchy sequence of elements of the field has a limit. The real numbers are the completion of **Q** with respect to the archimedian absolute value. An alternative definition of $\mathbf{Q}_p$ is as the completion of **Q** with respect to the non-archimedian (p-adic) absolute values.

This leads naturally to the study of $\mathbf{Z}_p$ and $\mathbf{Q}_p$ and various topological properties: compactness, local compactnes and so on. I also examied various forms of Hensel's Lemma, and some properties of local and global roots in **Q** and $\mathbf{Q}_p$. Finally I looked at finite field extensions of $\mathbf{Q}_p$. This required studying normed vector spaces in general. This took me to the end of the six weeks.

I found my summer scholarship both interesting and challenging. For those who enjoy pure mathematics I strongly recommend learning more bout p-adic integers and p-adic absolute values.

Thomas received an ICE-EM Vacation Scholarship in January 2005.
See www.ice-em.org.au/students.html#scholarship.