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Investigation of a new method of constructing packings of K_{6m+5}

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I spent my vacation scholarship extending an idea which I started investigating for a university subject in Semester 1, 2005.

A Steiner Triple System of order v (STS(v)), is a collection of triples chosen from a set of size v, such that each unordered pair of elements in the set occurs in precisely one triple. An STS(v) can also be thought of as a decomposition of the complete graph, K_v into triangles. A Steiner Triple System can only be created where $v \equiv 1$ or 3 (mod 6). Where $v \equiv 5$ (mod 6), a maximum packing of triples can be created, with a complete graph of order 5 left over.

There are existing constructions for a maximum packing; however my idea was an alternative way to construct a maximum packing of triples for $v = 6m + 5$, by extending an STS(6m + 3).

To do this, a particular configuration of triples was needed. A compound triangle is composed of 3 triples – where each pair of triples intersects in exactly one point. To be extended, the STS(6m + 3) needed to contain m compound triangles and one triple which covered all the points from 1 to v exactly once. It was then simply a matter of adding in the extra two points with each compound triangle and taking an alternate matching, and adding the two extra points in with the last triple to make the complete graph of order 5.

By the end of the scholarship I had proved that there was a STS(6m + 3) of every order (m an integer and $m > 1$) which could be extended to a maximum packing for $v = 6m + 5$. However I would like to go on to prove that any STS(6m + 3) can be used for this purpose.

Overall, I found the scholarship an enjoyable way to pursue a research project, and it has given me enthusiasm for further studies.

Nicole received an ICE-EM Vacation Scholarship in December 2005.
See www.ice-em.org.au/students.html#scholarships05