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MATHEMATICS

Elliptic Curves

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In my project I studied Elliptic Curves under the supervision of Dr Victor Scharaschkin. I read the book, *Rational Points on Elliptic Curves* by Silverman and Tate, and did many of the exercises.

Elliptic curves are curves of genus one with a point. They can be transformed into Weirstrass form, $y^2 = x^3 + ax^2 + bx + c$. They are important in Number Theory because they have a group law: if P, Q are points on C then, because C is cubic, the line joining P and Q intersects C in a third point (counting multiplicities). If we define P+Q to be the reflection of this point about the x-axis then one can show that (C,+) is an abelian group with identity O.

Much of my study was concerned with describing the group of rational points on an elliptic curve. The group can be shown to be finitely generated. This is important because it means that there is a finite generating set for the solutions to cubic Diophantine equations. The torsion of the group is relatively easy to find. However the rank is in general very difficult to find. There is a procedure to find the rank in “easy” cases (in some cases such as $y^2=x^3+17x$ it does not work.)

I also looked at a special case of the above: the congruent numbers problem (which asks which rational numbers are the areas of rational right angled triangles. It can be shown to be equivalent to determining whether elliptic curves of the form, $C : y^2 = x^3 - A^2x$ have positive rank. There have been some partial results.

Finally I read about elliptic curves over **C** in Silverman’s *The Arithmetic of Elliptic Curves*. This gave a function which parameterises the points over **C** (this makes it much simpler than over **Q**) and uses this to show some results like $E(\mathbf{C}) \cong (Z/nZ)^2$

I have found the project enjoyable as it has allowed me to study a very interesting subject in a more self-directed way than I would in coursework.

Sean received an ICE-EM Vacation Scholarship in December 2006.
See http://www.ice-em.org.au/students.html#scholarships2007