INTERNATIONAL CENTRE OF EXCELLENCE FOR EDUCATION IN MATHEMATICS

Saddlepoint Approximation of Lattice Random Variables

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My vacation scholarship project with Professor John Robinson was about studying saddlepoint approximation and its application to discrete random variables. First I was introduced to some important asymptotic results, including Edgeworth expansion for densities when a density exists, the saddlepoint approximation for densities and the indirect Edgeworth approximation. I showed these expansion and approximation can also be applied to discrete lattice random variables with slight variations. Then I looked at the paper 'Saddlepoint Approximation of the Two-sample Wilcoxon Statistic' and derived the conditional saddlepoint approximation to the Wilcoxon statistic to be

$$P(U=l)=\frac{\sqrt{pq}e^{NK(s,t)-s(l+m(m+1)/2)-tm}}{\sqrt{2\pi NK_{02}}\sigma_{st}}$$

where

$$\begin{aligned} W &= \text{two sample Wilcoxon statistic} \\ U &= -m(m+1)/2 \\ S_1 &= \sum_{j=1}^{N} jI_j, S_2=\sum_{j=1}^{N} I_j \\ K(s,t) &= \frac{1}{N}\log Ee^{sS_1+tS_2}=\frac{1}{N}\sum_{j=1}^{N}\log(q+pe^{js+t}) \end{aligned}$$

where s,t is the solution of

$$NK_{10}(s,t)=x+m(m+1)/2, NK_{01}(s,t)=m$$

and

$$\begin{aligned} \sigma_{st} &= K_{20}-K_{11}^2/K_{02} \\ K_{ij} &= K_{ij}(s,t)=\frac{\partial^{i+j}K(s,t)}{\partial s^i \partial t^j} \end{aligned}$$

Then this was generalized to the Kruskal-Wallis statistic, which is a k-1 dimensional generalization of the two-sample Wilcoxon statistic, for which I obtained:

$$P(\mathbf{S_1}=\mathbf{x}|\mathbf{S_2}=\mathbf{n})=\frac{e^{NK(\mathbf{s},\mathbf{t})-\mathbf{s}^T x-\mathbf{t}^T n}}{(2\pi)^{(k-1)/2}N^{(k-1)/2}\sqrt{detK''(\mathbf{s},\mathbf{t})/detK_{02}(0,0)}}$$

The next part of the project was about obtaining the tail probabilities of two-sample Wilcoxon statistic and hopefully the Kruskal-Wallis statistic with a relative error of order 1/N. The Lugananni-Rice and Barndorff-Nielsen approximation results for lattice random variables were used here when finding these tail probabilities.

Overall, I found this program a very valuable and rewarding experience. Through working on the project, I have gained not only new mathematical knowledge, but also the feel of what mathematical research will be like. This experience will definitely benefit my future studies and I would like to thank my supervisor Prof. John Robinson and AMSI/ICE-EM for providing me with the opportunity to undertake a research project such as this.

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See http://www.ice-em.org.au/students.html#scholarships2007