A Multiple Time Scale Analysis of Bursting In a Model of Pituitary Cells

Theodore Vo
The University of Sydney

The aim to understand how complex oscillatory patterns arise in models of relaxation oscillators motivated the study of a pituitary lactotroph model. This study was performed using a mixture of analytical (for which an assortment of literature was consulted) and numerical methods. The model (consisting of four variables) had two distinct manifestations, depending on the form of the fast potassium current in the presence of low doses of dopamine.

In either of its manifestations, the lactotroph model could be reduced to a 3D system. Using standard dimensional analysis techniques, the model was written in dimensionless form featuring one fast and two slow variables. The model was then put in the standard form of a singularly perturbed problem and examined on its two distinct timescales.

Geometric singular perturbation theory was used to split the model into two simpler subsystems (one a 2D subsystem, the other 1D) in the hope that the concatenation of the information from these limiting problems would illuminate qualitative features observed in the model. The critical manifold, the interface between which the limiting problems interact, was also calculated and its structure studied under parameter variations.

Numerical investigations of the full model and its associated limiting problems were conducted using the software packages XPPAUT and MATLAB. The construction of orbits via the limiting problems was accomplished and so the motivating oscillatory behaviour could be explained from a geometric viewpoint. Moreover, the limiting problems allowed the driving mechanisms behind each of the oscillatory patterns to be identified. Issues related to the numerics hindered the accuracy and reliability of the results in some minor cases.

It was found that under variation of the parameters, such as the timescales and maximal conductance of different channels, the model underwent a transition from oscillatory, featuring classical and non-classical relaxation oscillations and mixed mode oscillations, to excitable, with a stable equilibrium state.

Theodore received an AMSI Vacation Scholarship in December 2008
See: www.ice-em.org.au/vs09.html

The project was a valuable experience as it allowed me to explore the avenues of mathematical research and gave insight into what a potential honours year would involve. Many thanks go to the AMSI for awarding me the scholarship, to the School of Mathematics and Statistics at the University of Sydney for its support and above all, to Dr. Martin Wechselberger for his ceaseless encouragement, assiduous guidance and infinite reserves of patience.

Theodore received an AMSI Vacation Scholarship in December 2008
See: www.ice-em.org.au/vs09.html