# Cores of Strongly Regular Graphs

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Cameron and Kazanidis [1] proved that all rank-3 graphs (which are necessarily strongly regular) are either cores or have complete cores, and conjectured [2] that this is true of all strongly regular graphs. My research project focused on investigating this conjecture.

The point graphs of partial geometries are an important class of strongly regular graphs. Godsil and Royle [3] proved that Cameron and Kazanidis' conjecture holds for the point graphs of non-proper partial geometries whose only (s+1)-cliques are lines of the underlying partial geometry.

I began my project by reviewing relevant papers and background literature, and had the opportunity to attend the Group Theory, Combinatorics and Computation Conference held at the University of Western Australia in honour of Professor Cheryl Praeger's 60$^{th}$ Birthday, at which Cameron presented a lecture on this topic. I then attempted to generalise the results I'd examined to other families of strongly regular graphs.

I proved that Cameron and Kazanidis' conjecture holds for the point graphs of proper partial geometries whose only (s+1)-cliques are lines. This is a new result generalising Godsil and Royle's proof for the generalised quadrangle case.

As with the non-proper cases, my proof requires (s+1)-clique/line equivalence; however this is a far more restrictive condition for proper partial geometries. Godsil and Royle provided a condition to guarantee (s+1)-clique/line equivalence, but this is not satisfied for any of the known proper partial geometries [4]. Thus, (s+1)-clique/line equivalence must be established through another means (such as examining a graph by computer) before my result can be applied. Hence, while my project yielded a new theoretical result, I have no concrete examples of graphs to which it could be applied.

It would be desirable to find arguments that apply to general geometric graphs without the need for (s+1)-clique/line equivalence. However, the existing arguments from Godsil, Royle and myself are so fundamentally dependent on this condition that I don't believe they may be easily generalised in such a fashion.

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My project culminated with a successful presentation at the CSIRO's Big Day In. It has been a rewarding opportunity to gain insight into mathematical research and professional academia. The result I proved will be presented in a short paper.

**References**

[1] P. J. Cameron and P. A. Kazanidis, *Cores of symmetric graphs*. Journal of the Australian Mathematical Society 85, 2, 2008, 145-154.

[2] Through personal communication with my supervisor, A/Prof. Gordon Royle.

[3] C. Godsil and G. F. Royle, *Cores of Geometric Graphs*. To appear in Annals of Combinatorics, 2008

[4] J. A. Thas, *Partial Geometries*. In Handbook of Combinatorial Designs, C. J. Colbourn and J. H. Dinitz, Eds. Chapman & Hall/CRC, Boca Raton, FL, 2006, 560.

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