Branching Processes

Leon Todorovich
School of Mathematics and Statistics
University of Western Australia

Branching processes are stochastic models of population growth with prominent applications in biology and physics. Originally introduced by Bienayme in 1856, and independently by Galton and Watson in 1873, limit theorems for the so called discrete time and state Galton-Watson process (or Simple Branching Process) were first derived by Steffenson in 1930.

As part of this project I investigated limit theorems for the continuous time/discrete state analogue of the Galton-Watson process, the Markov Branching Process, as well as for the continuous time and state version, the Continuous Branching Process. The critical case, where the mean per capita reproduction rate m=1, is of specific interest to biological population models and is explored in depth by Pakes (2008), where a unified treatment of limit theorems allowing for extremely infinite offspring variance is given, which completely generalises the 'classical' limit theorems which make additional moment and distributional assumptions.

As an introduction to the project I read several textbook accounts, especially those of Jagers (1997) and Athreya & Ney (1972), in order to become acquainted with the Simple Branching Process and Markov Branching Process. From there I read two papers by my supervisor, Tony Pakes, one in relation to critical limit theorems for the Markov branching Process and another (as yet unpublished) relating to limit theorems for the Continuous Branching Process.

Although my attempts to provide proofs for Continuous Branching Process theorems using the approaches for the Markov Branching process were not successful, I felt that by having to understand the underlying concepts related to Levy processes, infinite divisibility, stable laws and functions of regular variation that the project was especially beneficial in preparing me for the sort of work I can expect to do in honours. Working closely with an expert in the field and focusing on a specific are of research was a very valuable experience, as was the exposure to other students and academics I gained by attending the Big Day In.